Abstract—Recently we proposed so-called bi-SQUID based on a 3-junction SQUID circuit capable of providing highly linear voltage response. In this report, we present the experimental evaluation of series arrays of 20 and 128 bi-SQUIDs fabricated with a 4.5 kA/cm² Nb process as well as a prototype of an active electrically small antenna based on series array of 12 bi-SQUIDs. Both the origins of imperfections of the observed response linearity and the possible ways of the linearity improvement are discussed.

Index Terms—Josephson junctions, SQUID, bi-SQUID, SQIF, voltage response, high linearity, electrically small antenna.

I. INTRODUCTION

The electrically small antenna (ESA), i.e. antenna with a size that is much less than one wavelength, is very attractive for many applications due to the small antenna size and the wide bandwidth [1]-[4]. But in some cases (loop type antennas) the small size results also in a low radiation resistance. As far as one tries to make the radiation resistance much more than the antenna conductor loss resistance to increase antenna sensitivity, the use of superconductors instead of the metal conductors substantially improves the antenna characteristics [4]-[7], but usually requires thorough matching with load impedance [8], [9].

Next critical step is the development and implementation of an active ESA. This promises a substantial improvement of all characteristics of the antenna including solution of the matching problem.

There are two possible approaches to design of the active electrically small antenna. The first approach is based on combination/integration of the metal-conductor or superconductor ESA with the small high-sensitive broadband amplifier, for example the SQUID or SQUID-array based superconductive amplifier [10]. But such amplifier ought to have wide dynamic range and high linearity to provide high performance of the antenna.

The second approach to the active ESA design is the implementation of an active Josephson-junction array network capable of providing wide dynamic range and highly linear voltage response directly as antenna. But not every such network can be successfully used for this purpose. Initial homogeneity of the incident electromagnetic wave is always disturbed along network due to Meissner effect in superconductors. Therefore this will result in the degradation of the network response linearity if this linearity is resulted from proper contributions of all cells in the network. Such network needs a specially designed input control line to realize the required pattern for input magnetic signal to the network [11]. Therefore the network can be used only as an amplifier connected to a superconducting ESA.

However, if the network consists of the cells each providing linear voltage responses, the accrued inhomogeneity of the magnetic field component and hence the difference in fluxes applied to the cells does not affect the network response linearity. Such a cell called bi-SQUID was recently proposed [12], [13]. We believe that the series array of the bi-SQUIDs can be used as an active electrically small antenna.

II. Bi-SQUID

We modified conventional dc SQUID by adding a nonlinear inductive element shunting the linear inductance of the loop coupling RF magnetic flux into the SQUID (see Fig. 1). This nonlinear element modifies the nonlinear transfer function of the SQUID to produce higher linearity transfer function, thus
increasing the utility of the device as a linear sensor or amplifier. The nonlinear inductive element is a Josephson junction that remains in its superconductive state during operation. The nonlinear shunt inductance is the Josephson-junction inductance:

\[ L_J = \Phi_0 \left( 2\pi I_3 \sqrt{1 - i^2} \right), \]  

where \( i = I_{sh}/I_3 \) is the normalized current passing through the shunt junction with critical current \( I_3 \). The effective loop inductance is a parallel combination of the main inductance \( L \) and Josephson inductance \( L_J \). The additional junction and main inductance form a single-junction SQUID. In such a wave, the modified dc SQUID can be called bi-SQUID. Optimal parameters of the one-junction SQUID can be stated approximately as \( I' = 2\pi I_3/L\Phi_0 = l/I_3/C \approx 1 \), where \( l = 2\pi I_3/L\Phi_0 \) is normalized inductance of the base symmetric dc SQUID with critical current \( I_C \) of its Josephson junctions, \( \Phi_0 = h/2e \) is magnetic flux quantum [12]. This means that the normalized critical current of the third junction \( I_{sh}/I_3 \), should be decreased with increase in the normalized inductance \( l \) of the base dc SQUID.

When no bias current is applied to dc SQUID, differential phase \( \phi = \varphi_1 - \varphi_2 \), where \( \varphi_1, \varphi_2 \) - Josephson-junction phases) behavior is similar to the one for Josephson-junction phase in a one-junction SQUID. But in resistive state the dc SQUID differential phase manifests a radically different behavior. Numerical simulation shows that the mean value of the differential phase replicates the applied magnetic flux: \( \phi \approx \varphi_2 = 2\pi \Phi_0 / \Phi_0 \) (see curve 1 for \( l = 1 \) and curve 2 for \( l = 10 \) in Fig. 2). When we add to this dc SQUID the 3-rd Josephson junction, the differential phase behavior pattern changes radically. In case of bi-SQUID, the behavior of the differential phase mean value is about the same for both superconducting and resistive states (see Fig. 2, curves 3 and 4). This fact means that in resistive state of bi-SQUID, its one-junction interferometer provides just the same nonlinear transformation of the applied low-frequency input signal to the differential phase mean value \( \dot{\phi} \) as in superconductive state.

Fortunately, as it follows from numerical simulations, such nonlinear transformation at \( I' \approx 1 \) is just the one needed to linearize voltage response.

III. EXPERIMENTAL EVALUATION

Series arrays of bi-SQUIDs including prototype of active electrically small antenna where designed and fabricated using a 4.5 kA/cm² Nb HYPRES process [14]. We used separated sections of double ground planes for each bi-SQUID in the serial arrays to eliminate shunting effect of the stray capacitances which are characteristic for the standard circuit designs with two superconducting ground planes [11].

Fig. 3 shows photograph of the fabricated and tested prototype of an active ESA based on serial array of 12 bi-SQUIDs. Large superconducting loops to sense magnetic component \( B \) of an incident electromagnetic wave were included in the design of bi-SQUIDs. Schematic of the bi-SQUID cells is presented in Fig. 1b. This is a bi-SQUID
placed between two ground planes (shaded area) and the outside input loop. To perform the initial test of such an antenna, a large-size strip coil was introduced in the fabricated chip to excite a low-frequency or dc magnetic signal.

Fig. 4 shows voltage response of the antenna prototype with 50 Ohm load. The bi-SQUID array is biased with current \( I_b \) which is slightly more than the bi-SQUID critical current. Existence of small hysteretic parts on top of the presented voltage response evidences that parameter \( l^* \) (it refers to parallel connection of \( L \) and \( L_{ex} \)) exceeds 1. The measured transfer factor of the antenna prototype is as high as 50 V/T. The increase in number of bi-SQUIDs in antenna prototype will result in proportional rise of the transfer factor.

All the other tested serial arrays of bi-SQUIDs had specially designed control strip lines intended for the magnetic signal application to the bi-SQUIDs. Series of the measured voltage responses of serial array of 20 bi-SQUIDs with the bias current increase are presented in Fig. 5. The normalized inductances \( l^* \) of the one-junction interferometer loops were about 2 and therefore the observed voltage responses show hysteretic behavior when the applied magnetic flux becomes close to \( \Phi_0/2 \). Fig. 6 shows voltage response of large serial array of 128 bi-SQUIDs with normalized inductance of the one-junction interferometer loop \( l^* \approx 1.2 \). The array is biased by the current \( I_b \) close to critical current \( 2I_c \) of the bi-SQUIDs.

Fig. 7. The measured voltage response of serial SQIF consisting of 40 bi-SQUIDs. The SQIF is loaded at 50 Ohm.

The measured voltage response amplitude is as high as 7 mV. One of the fabricated and evaluated arrays is a serial SQIF [15] consisting of 40 bi-SQUIDs with widely different periods of their voltage responses. The different periods were realized by a 30-times varying of the mutual inductances between the input strip line and the loops of bi-SQUIDs. Voltage response of the SQIF loaded at 50 Ohm shows a single central peak with amplitude 1.75 mV. As seen in Fig. 7, response peaks are quite linear.

IV. DISCUSSION

According to numerical simulations of a bi-SQUID with vanishing thermal noise, its voltage response ought to approach a triangular form at critical current biasing \( I_b = 2I_C \). But the measured voltage responses including the one for SQIF show smoothed corners. Such a smoothing may result from thermal smoothing of Josephson-junction critical current. This effect becomes more pronounced with decrease in the junctions critical currents. At high critical currents of the junctions used in the evaluated circuits (\( I_C = 125 \mu A \)) this effect is negligibly small and the observed smoothing might follow from technological spread in the junction critical currents [14]. Fig. 8 shows voltage responses of bi-SQUID calculated with biasing currents \( I_b = 2I_C \) and \( I_b = (1\pm0.025)2I_C \). The conditions for the
linear triangular voltage response are very sensitive to deviations of the biasing current from the bi-SQUID critical current $2I_C$.

To depart from the singular point in the current biasing, one can turn to differential connection of bi-SQUIDs. Our preliminary calculations show that the bi-SQUID voltage response can approach a parabolic shape with the biasing current decrease down from critical current value. In this case differential voltage response should approach a highly linear shape \[16\].

One can also suggest the design and integration of a differential circuit with two bi-SQUIDs into one cell providing a linear voltage response. Fig. 9 presents a possible schematic of the cell aimed at high-frequency applications. In this cell, both the dc bias current $I_B$ and the input rf signal $\delta B$ (induced by current $I_{M,RF}$) are applied to the bi-SQUIDs with opposite signs, while the dc magnetic biasing $B_{DC}$ (induced by current $I_{M,DC}$) is done equally. The high frequency output voltages of the two bi-SQUIDs are added to the overall high frequency output voltage by means of the connection via the capacitors.

The integrated cells can be connected in series to increase dynamic range and output signal amplitude. Such a serial array of the cells each providing highly linear voltage response could be used to design the active electrically small antennas.

V. CONCLUSION

Series arrays of bi-SQUIDs including prototype of an active electrically small antenna where designed, fabricated and tested. Voltage response with amplitude as high as 7 mV was observed for the serial array of 128 bi-SQUIDs. The measured transfer factor of the antenna prototype based on serial array of 12 bi-SQUIDs is as high as 50 V/T. The further increase in the number of bi-SQUIDs will result in a proportional rise of the transfer factor.

The measured voltage responses of the serial arrays of bi-SQUIDs show significant improvement in the response linearity in comparison with arrays of conventional dc SQUIDs. But the linearity improvement is also quite sensitive to spread in Josephson-junction critical currents resulting in essential smoothing of the response corners.

Further increase of linearity in bi-SQUID array circuits can be obtained with the use of a differential connection and the suggested integrated cells with linear response.

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REFERENCES