Resonance Features of Coupled Josephson Junctions in High Temperature Superconductors

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Abstract - Intrinsic Josephson junctions in high temperature superconductors have a good perspective for different electronic devices (in particular, for the quantum voltage standard) operated at relatively high temperature and over a wide frequency range. We investigate the parametric resonance in the intrinsic Josephson junctions and creation of the longitudinal plasma wave. The charge oscillations in the superconducting layers at different values of bias current are studied. We demonstrate that the resonance characteristics might be used for the determination of the system’s parameters.

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I. INTRODUCTION

The single Josephson junction demonstrates an interesting physics and variety of applications. From the point of view of practical applications, a great number of junctions is often needed. For example, in a quantum voltage standard, thousands of junctions in series are
required to obtain Shapiro steps at sufficiently high voltages up to 10 V [1]. The intrinsic Josephson junctions (IJJ) within the high-$T_C$ superconductors offer a new way to realize a large number of junctions in a very compact way [2,3]. As was mentioned in Refs.[3,4], compared to the well-developed low-$T_c$ dc voltage standards, a few advantages can be found using IJJs: (i) high operation temperatures, up to temperatures near 50 K; (ii) operation at higher frequencies up to some THz; and (iii) high density of junctions due to naturally compact atomic scale, which may simplify the instrumentation significantly. But it has to be investigated if the extremely thin electrodes of the IJJs and their strong interaction results in stability and noise effects disturbing the function of such voltage standards.

The system of IJJs in high temperature superconductors are formed from the atomic scale superconducting layers (S-layers) [5], characterizing by modulus of order parameter $\Delta$, and its phase $\theta$ (see Figure 1a). The thickness of S-layers is comparable with the Debye screening length. Because of it, the electric charge does not screen in S-layers perfectly and it leads to the capacitive coupling between the junctions [6].

The phase dynamics of IJJ without magnetic field is determined by capacitive coupling and the system of IJJ is described by capacitively coupled Josephson junctions (CCJJ) model [6] or model with the diffusion current (CCJJ+DC) [7,8]. We studied the multiple branch structure of the current-voltage characteristics (CVC) of IJJ and showed that branches have a breakpoint (BP) and some breakpoint region (BPR) before transition to another branch [8]. The BP current is determined by the creation of the longitudinal plasma waves (LPW) with a definite wave number, which depends on the coupling and dissipation parameters, the number of junctions in the stack, and the boundary conditions.

In [9,10] a correlation between the charge dynamics on the S-layers and features of CVC was established and we showed that the breakpoint on the outermost branch of CVC is related to the parametric resonance in the system of Josephson junctions and creation of longitudinal plasma wave (LPW). Experimental manifestation of the breakpoint and the BPR [11] in CVC of Bi$_2$Sr$_2$CaCu$_2$O$_y$ stimulated new investigations in this field. The observed powerful coherent radiation from the stack of IJJs in layered superconductor BSCCO makes this system very promising for other different applications [12].

II. PARAMETRIC RESONANCE

As was shown in Ref. [6], the system of equations for CCJJ has a solution corresponding to the LPW propagating along the $c$ axis. At the condition $\omega_j = 2\omega_{LPW}$ the parametric resonance is realized in the coupled system of JJ. In Figure 1(right) is shown how the charge appears in S-layers with approaching the resonance point. First the charge appears...
as fluctuations, then the islands form (IR) which are transformed to the regions with alternating amplitude (AAR) and growing region (GR).

![Diagram of Josephson junctions]

**Fig.1.** Left - The schematic setup of intrinsic Josephson junctions. Right – Charge on the superconducting layer as a function of time. The thick curve shows the variation of bias current.

Using Maxwell equation, we express the charge density in the S-layer by the voltages in the neighbor insulating layers \( Q_i = Q_o \alpha (V_i - V_{i-1}) \), where \( Q_o = \varepsilon \varepsilon_0 V_0 / r_0^2 \), \( V_0 = \hbar \omega_p / 2e \), \( \omega_p \) is plasma frequency and \( r_0 \) is Debye screening length. We can estimate the value of electric charge in the superconducting layer. Using \( \varepsilon_0 = 0.0885 * 10^{-11} \) F/m, \( e / \hbar = 2.4 * 10^{14} \) 1/V * s and consider \( r_0 = 3 * 10^{-10} \) m, \( \varepsilon = 25 \), \( \omega_p = 10^{12} \) 1/s, we get \( V_0 = 2 * 10^{-3} \) V and \( Q_o = 5 * 10^8 \) K/m\(^3\). So, at \( Q = Q_o \) for the superconducting layer with area \( S = 1 \mu m^2 \) and thickness \( d = 3 * 10^{-10} m \), the charge in the superconducting layer is equal to \( 1.5 * 10^{-15} K \). This value is not high enough, but the charge dynamics on the superconducting layers determines the features of current voltage characteristics of the coupled Josephson junctions.

In the stack with even number of JJ at \( \alpha = 1 \) and \( \beta = 0.2 \) and periodic BC, the LPW with wave number \( k = \pi \) is created. In this case the wavelength of the LPW \( \lambda = 2 \) commensurate with the length of the stack \( Nd = n \lambda \), where \( d \) is a period of lattice and \( n \) is an integer, and it leads to the absence of the fine structure in the CVC. So, we can observe the "pure" parametric resonance in this case. It’s interesting to investigate the parametric resonance and charge creation and the nucleation of the LPW in the stack without any other effects.

The increase of charge in the GR follows to the exponential low as it should be at the resonance condition. In Figure 2 (left) we show the charge-time dependence for the first S-layer in the growing region. The y-axis demonstrates it in the logarithmic scale. We see that the increase of charge follows to the exponential law in some part of the growing region only.
Position of the breakpoint on the outermost branch of CVC corresponds to the end of this exponential growth.

Figure 2 (right) shows the charge oscillations in the S-layer in the transition region (from the growing region to the region corresponding to another branch). The maximal value of the electric charge in the end of growing region is around $\frac{Q}{Q_0} \approx 1$.

The part of 10 IJJ CVC is presented in Figure 3a. It shows the outermost branch R and the transition to the branch with one junction in oscillating state O(1). The inset shows the...
enlarged part of the CVC demonstrating the absence of the fine structure [9,10] in R-branch and very sharp transition to the wide BPR on the branch O(1).

In Figure 3 (b) we show together the time dependence of the charge in the first layer and its CVC. The left y axis shows the charge, the lower x axis shows time; the upper x axis belongs to the current, and right y axis belongs to the voltage. The inset demonstrates the onset of the growing region of the charge amplitude in S-layer. We see that the BP position does not coincide with the onset of the growing region.

III. TIME DEPENDENCE

Solution of the system of dynamical equations for the gauge-invariant phase differences between S-layers gives us the voltages in all junctions in the stack, and it allows us to investigate the time dependence of the charge on each S-layer. The "time dependence" consists of time and bias current variation.

Let us demonstrate in detailed the formation of the LPW in time. Figure 4 left shows the charge oscillations in two first S-layers in the stack with 10 IJJ at different values of bias current. As we mentioned above, first the fluctuations of the charge in S-layers with the value about $10^4 Q_0$ are observed in Figure 4 left (a,b). Positive and negative charge appears in a irregular way. We show the charge in first layer by solid line and the charge in the second S-layer by dashed line. Then the LPW is formed and we call the corresponding point in CVC as the breakpoint B. In during some period of time the amplitude of LPW is changing non-regular way. But we should stress that the amplitude value of the charge in different S-layers is practically the same. Then the amplitude of the charge oscillations starts increase continuously. In Figure 4 left (c) we show such increase of the charge value. In the small current interval (0.5769, 0.5767) it grows by 5 orders. In Figure 4 left (c) we demonstrate the charge oscillations in the beginning of the region with growing amplitude. Figure 4 left (d) demonstrates the oscillations in the transition region. We show a transition point to another branch at $I_c=133755$ by arrow. So, we may distinguish clearly three different stages in LPW formation: fluctuation (LPW in short time interval on the noise level); island (oscillations with alternating amplitude, exceeding noise level); growing (oscillations with growing amplitude).

The examples of the charge distribution along the stack with 10 coupled JJ are presented in Figure 4 (right). We see that the distribution of charge corresponds to the modulated LPW with $k=\pi$ enough good even in the island region. Figure 4 right (a) shows the charge distribution along the stack with 10 IJJ at $I=0.5773 \, \text{. Solid line (green online) shows the wave } 10^4 \sin \pi z \, \text{. Figure 4 right (b) demonstrates the charge distribution along the stack at } I=0.577. \text{ The amplitude of the charge oscillations here grows by 7 orders of value in the small current interval (0.5769, 0.5767).}
IV. DETERMINATION OF SYSTEM’S PARAMETERS

The study of the breakpoint features in the CVC of IJJ allows us to develop a new method for determination of the system’s parameters. In the case of coupled system of junctions, the parameter $\beta$ can not be determined usual way (by the return current), because the return current depends now on two parameters, as $\beta$ and $\alpha$. The CVC of the stacks with odd number $N$ of IJJ at periodic boundary condition demonstrates the same behavior for the $I_{bp}$ and BPR width $w_{bp}(N)$ as in the non-periodic case, but for the stacks with even $N$ the value of the $I_{bp}$ does not depend on the $N$ and the BPR is absent for these stacks [8].

We may estimate the value of $\beta$, using the results of these simulations of CVC for stacks with different number of junctions. At $\alpha = 1$, $\beta = 0.2$ and periodic boundary conditions the stacks with even number of junctions have the same value of $I_{bp}=0.576$, because the same $\pi$-mode is created at the breakpoint. For example, for $N=10$ we get the value of $V_{bp}=28.595$, found from the simulation, and the value of $\beta = 0.2014$, calculated at $I_{bp}=0.576$ by the formula $\beta = N \times I_{bp} / V_{bp}$ which follows from the breakpoint position in the CVC.

The absolute error (absolute accuracy) in this calculation consist of 1-2 percents of the $\beta$ value. The estimation gives the conservative value. The same order of the absolute

Fig. 4. Left - The charge oscillations in the first and second S-layers in the stack with 10 IJJ at different bias current values: (a) $I=0.59$, (b) $I=0.5772$, (c) $I=0.5770$, (d) $I=0.5767$. Right - (a) Charge distribution along the stack with 10 IJJ at $I=0.5773$. The solid line shows the wave $10^{-8} \sin \pi z$. (b) Charge distribution along the stack with 10 IJJ at $I=0.577$. 
error we have for the stacks with odd number of junctions. Particularly, for N=11, \( I_{sp} = 0.5721 \) and \( V_{sp} = 13.794 \), and we have \( \beta = 0.2017 \).

V. CONCLUSIONS

We have shown that the parametric resonance and the creation of the longitudinal plasma wave in coupled Josephson junctions play an important role and determine the features of current voltage characteristics of this system. The investigation of the resonance characteristics allow one to develop a new method for determination of system parameters. The influence of the parametric resonance and the charge oscillations on the behavior of electronic devices described in Refs. [2,3] like mixers, field-effect transistors and voltage standards realized with IJJs has to be studied in more detail to avoid problems in application of such devices.

REFERENCES