Fast method to quantify the collective magnetization in superconducting magnets
Emmanuele Ravaiolī1,2, Bernhard Auchmann1, and Arjan P. Verweij1

Abstract—The magnetization of the superconductor is one of the most important parameters determining the field quality of accelerator magnets. A fast method to quantify the magnetization effect in an entire magnet was developed at CERN based on a voltage-current measurement during a powering cycle. The collective magnetization includes the effect due to hysteresis losses in the magnet superconducting filaments, coupling losses in the magnet conductor, and magnetization of the iron yoke. It is calculated by means of an energy balance between the work done by the power converter and the change of magnetic energy in the system. Also the energy dissipated at any time is calculated. In the magnet test facility at CERN, LHC dipole magnets have been cycled between ±600 A with a ramp-rate of 10 A/s. The magnetization curves deduced from these measurements show a good precision and high reproducibility, mainly due to the high precision of the power converter and the current-measurement system. The results have been compared with numerical simulations performed with the computer code ROXIE. The proposed test method can be applied to any type of magnet, is rather easy and fast, and is therefore interesting for checking the reproducibility of the magnetization among a series production of magnets.

Index Terms—Accelerator magnets, Magnetization, Measurement techniques, Superconducting magnets.

I. INTRODUCTION

The magnetization of the superconductor is one of the most important parameters determining the field quality of accelerator magnets. Usually the magnetization is measured on single superconducting strands or cables [1-5], and the effect on the field quality is then calculated using numerical software [6]. Furthermore, the field quality can be measured using pick-up coils located in the aperture of the magnet [7], and then compared to the expected calculated field.

A fast method to quantify the magnetization effect in an entire magnet was developed at CERN. The voltage across a magnet and the current flowing through it are measured during a powering cycle. The current in the magnet is cycled by means of a 600 A 10 V four-quadrant power-converter. The measurement system is composed of a high-precision direct current-current transformer (DCCT) [8], a NI-USB-6251-BNC measurement card, and a personal computer.

One channel of the measurement card acquires the differential signal between the voltage taps placed across the magnet, whereas a second channel records the signal measured by the DCCT, corresponding to the current flowing through the magnet. The acquisition frequency was set to 2 kHz.

B. Calculation of the Equivalent Strand Magnetization

For an electromagnetic system the work done by the system \(dW_{PC}\) must equal the variation of magnetic energy in the system \(dE_B\) [9],

\[
dW_{PC} = dE_B = \iiint_V B \cdot dB \, dV = \mu_0 \iiint_V H \cdot dB \, dV + \mu_0 \iiint_V H \cdot dM \, dV, \tag{1}
\]

where \(B=\mu_0(H+M)\) is the magnetic induction [T], \(H\) is the magnetic field [A/m], \(M\) is the magnetization [A/m], \(\mu_0\) is the vacuum magnetic permeability and equals \(4\pi \times 10^{-7}\) T m/A, and the integrands are integrated over a suitably chosen volume. Given a set of measurement data \((U_i, I)\), the work done by the iron yoke. Also the energy dissipated at any time is calculated. The description of the method is given in Section II.

In order to validate the method and assess its precision, tests have been performed in the magnet test facility at CERN. A LHC twin-aperture dipole magnet has been cycled between ±600 A with a 600 A 10 V four-quadrant power converter. The results have been compared with numerical simulations performed with the computer code ROXIE [6].

The proposed methodology can be applied to any magnet type, and since it is easy and fast it is a useful method for checking the reproducibility of the magnetization effect among a large series of magnets, especially if the existing alternative magnetic measurements are time consuming. The calculated magnetization represents the collective magnetization as a function of the current in the magnet, but it does not give information about the variation of the field quality in different positions of the magnet.

II. DESCRIPTION OF THE METHOD

The method relies on the direct measurement of the voltage \(U\) across the magnet and of the current \(I\) flowing through it during a powering cycle.

A. System Setup

The current in the magnet is cycled by means of a 600 A 10 V four-quadrant power-converter. The measurement system is composed of a high-precision direct current-current transformer (DCCT) [8], a NI-USB-6251-BNC measurement card, and a personal computer.

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\[
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\]
system, i.e. by the power converter, can be discretized as
\[ \Delta W_{PC,i} = UdI dt \approx UI_i \Delta t, \]  
where the index \( i \) refers to the \( i \)-th measured point, and \( \Delta t \) is the time between two measured points, i.e. the inverse of the measurement frequency.

The first term on the right side of (1) represents the change of magnetic energy due to a change of the magnetic field. Its change during \( \Delta t \) for a linear inductor can be expressed as
\[ \Delta E_{H,i} = \mu_0 \int \int H_i \Delta H_i dV = L_H I_i \Delta I_i, \]  
where \( L_H \) is the inductance of the inductor only due to the magnetic field [H]. \( L_H \) depends on the geometry of the inductor and can vary with the current due to the saturation of the iron yoke surrounding the magnet. However for small currents it can be approximated as the average value of the differential inductance of the inductor \( L_d \) during a powering cycle, since the magnetization-related contributions to \( L_d \) during two symmetric positive and negative ramps compensate each other. \( L_d \) can be calculated at any time as
\[ L_{d,i} = \frac{U}{(\Delta I / \Delta t)} = \frac{U_i}{(I_i - I_{i-1})} \Delta t. \]  

The second term on the right side of (1) represents the energy lost due to magnetization effects. For a superconducting magnet, its variation during \( \Delta t \) is the sum of a contribution due to the hysteretic magnetization change in its superconducting filaments \( \Delta E_{M,sc,i} \), and a contribution due to other magnetization changes \( \Delta E_{M,ot,i} \):
\[ \Delta E_{M,sc,i} = \mu_0 \int \int H_i \Delta M_i dV = \mu_0 V_{sc} f_{sc,i} I_i \Delta M_{sc,i} + \Delta E_{M,ot,i}, \]  
where \( \Delta M_{sc,i} \) is the hysteretic volume of the conductor, thus replacing \( V_{sc} \) in (9) with the total volume of the conductor \( V_c \). Equation (9) contains only measured values of \( U, I, \) and \( \Delta t \), the parameters \( L_H, f_{sc}, \) and \( V_{sc} \), depending on geometric properties of the magnet and usually well known, and \( M_{col,0} \). The latter is zero for the first magnetization cycle performed after the cooling down of the magnet. For subsequent cycles, it can be taken as the last calculated value of \( M_{col} \) in the previous cycle, or estimated considering the magnetic history of the magnet.

III. APPLICATION OF THE METHOD

A. Test Magnet

In order to validate the technique presented in Section II, voltage-current measurements were performed at a temperature of 1.9 K in the CERN magnet test facility. The measured magnet (MB1089) is a LHC twin-aperture main dipole magnet [12], with \( V_c=0.0997 \) m³ and \( V_{sc}=0.0354 \) m³. It is composed of two different Nb-Ti cable types: in the inner, higher field region the cables are thicker and their superconducting filaments have a diameter \( d_f \) of 7 \( \mu \)m; in the outer, lower field region \( d_f=6 \) \( \mu \)m. \( f_{sc} \) was evaluated with numerical simulations and equals 455 T/A. For the tested magnet \( L_H \) equals 98.1 mH, calculated as the average \( L_d \) within a \( \pm600 \) A powering cycle (see Section II.b).

B. Calculated Collective Magnetization

The test presented in this paper featured two powering cycles with constant ramp-rate. Fig. 1 shows the \( U \) and \( I \) signals recorded during Cycle 1 and 2, together with the calculated ramp-rate \( dI/dt \). It can be observed that during the ramps \( dI/dt \) remained constant whereas the voltage across the magnet varied. This is due to the fact that the differential inductance is not constant throughout the cycle.

FIG. 1 HERE

The calculated differential inductance as a function of the magnet current is shown in Fig. 2. The initial value of \( L_d \) was less than 85% of \( L_H \). This is due to the residual positive magnetization within the magnet, which gave a negative contribution to \( L_d \). The two cycles presented in this paper were preceded by a negative ramp of the current, thus initially they both present \( L_d>L_H \). This phenomenon is the reason why the inductance of the LHC main dipole magnets measured with a frequency-sweep at very low current was often found to be smaller than the expected value [13].

FIG. 2 HERE
Fig. 3 shows the calculated collective magnetization within the magnet conductor as a function of the current \( I \) during Cycle 1 and 2. The average hysteretic magnetization within the magnet superconducting strands \( M_{sc,ROXIE} \), modeled with a numerical simulation carried out using the program ROXIE and averaged over the magnet strands, is presented as well. Considering (8), one can see that the difference \( M_{col,M_{sc,ROXIE}} \) gives an indication of the value of \( E_{H,ot} \). This value is affected by a large number of errors though, coming from the voltage-current measurement error, the inaccuracy of the ROXIE magnetization model at very low fields, end effects in the two magnet extremities, and the imprecision in the calculation of \( L_{Jos}, f_{sc}, \) and \( V_c \).

The magnetization within the conductor in Cycle 1 and 2 is mainly due to hysteretic effects. In fact, the contributions to the strand magnetization due to inter-filament and inter-strand coupling-losses \( M_f \) and \( M_L \) were estimated to be very small as compared to the hysteretic magnetization. In fact, from [10]

\[
M_f \approx \left( \frac{l_w}{2\pi} \right)^2 \frac{1}{\rho_{eff}} \frac{dB}{dt} \approx \left( \frac{l_w}{2\pi} \right)^2 \frac{1}{\rho_{f}} \frac{f_{sc}}{dt}, \quad \text{[A/m]} \tag{10}
\]

where \( l_w \) is the cable twist-pitch [m], and \( \rho_f \) its effective transverse resistivity [\( \Omega \)m]; and from [11]

\[
M_L \approx 8.49 \times 10^{-3} \frac{N_s}{R_c} \frac{(N_s^2 - N_s^3)}{B_2} \frac{dB}{h}, \quad \text{[A/m]} \tag{11}
\]

where \( N_s \) is the number of strands in the cable, \( w \) and \( h \) the broad and narrow dimensions of its cross-section [m], \( R_c \) the contact resistance between two strands [\( \Omega \)], and \( B_2 \) the magnetic induction perpendicular to the cable broad face [T].

During Cycle 1 and 2 \( \frac{dI}{dt} = \pm 10 \text{ A/s} \) and \( \mu_0 M_f \) \( \mu_0 M_L \) are estimated to be smaller than \( \pm 0.1 \text{ mT} \) both in the inner and outer strands.

The average magnetization in the conductor is thus the result of the hysteretic magnetization among strands which behave differently. In fact, in the strands located in high-field regions of the magnet the magnetic field changes more quickly than in those located in the low-field regions. Besides, since the critical current density in the superconducting strands \( J_c \) is higher at lower field, the penetration field \( H_p = J_c(B) dJ_c/\pi \) is larger in the low-field strands. When in a strand \( H > H_p \), the magnetic field fully penetrates its filaments and \( M_{sc} \) in the strand reaches the saturation value defined by [10]

\[
M_{sat} = \frac{2\lambda d_J f_c(B)}{3\pi}, \quad \text{[A/m]} \tag{12}
\]

where \( \lambda \) is the fraction of superconductor in the magnet strands. Saturation curves for the two strand types in the magnet calculated with the \( J_c(B) \) fit proposed in [14] are shown in Fig. 4.

At the beginning of Cycle 1 most high-field strands were fully magnetized due to the magnetic field remaining from a precedent magnetization cycle, whereas most low-field strands had a positive but unsaturated magnetization.

As the current increased, the magnetic field in the strands increased and the magnetization-change was negative in order to oppose to the field-change. The strands saturated when their magnetic field reached their respective \(-H_p\). Assuming \( H = H_p \) in all the strands at the beginning of the cycle, one can assume that any strand saturated after a field-change of about \( 2H_p \), and then estimate the number of saturated strands during the cycle by calculating \( H \) and \( H_p \) of each strand. About 30\% of the strands saturated below 200 A, 65\% below 400 A, and 85\% below the maximum current of the cycle 600 A. These figures should be increased since some strands started the cycle with \( H < H_p \) and needed a smaller field-change to saturate. The average magnetization in the strands at 600 A approached the value of \( M_{sat} \).

During the second half of Cycle 1, the current decreased and most of the high-field strands quickly re-saturated to a positive value of magnetization. However, in the low-field strands \( H_p \) was larger, and the field-change smaller and too little for fully saturating the filaments. At the end of Cycle 1, when the current reached zero, about 60\% of the strands were saturated.

Most of the low-field strands that did not reach saturation during the negative ramp of Cycle 1 saturated when the field was further decreased, at the start of Cycle 2. This is the reason why the calculated magnetization curve has a positive peak at a negative current instead of at zero. When the current reached the minimum value -600 A, nearly all the strands were saturated.

The behavior of the strand magnetization during the positive ramp from -600 to 600 A was similar to that during the negative ramp: the high-field strands saturated more quickly than the low-field ones, about 60\% of the strands saturated before reaching positive current, and at 600 A nearly all the strands had reached saturation.

During the final current ramp back to zero which concluded Cycle 2, the magnetization effects closely replicated those occurred during the ramp 600-0 A of Cycle 1.

It is likely that the area of the loop defined by the evolution of the quantity \( M_{col,M_{sc,ROXIE}} \), related to \( E_{H,ot} \) during Cycle 2 is linked to a hysteresis loop due to magnetization of the iron yoke surrounding the magnet.

The explanation for the two peaks of \( M_{col,M_{sc,ROXIE}} \) at low current \( (|I|<150 \text{ A}) \) seems to be related to the ROXIE magnetization model which is less reliable at very low fields.

**FIG. 3 HERE**

Fig. 4 shows \( W_{PC} \), \( E_{H} \), and \( E_{ot} \) defined as the discretized integrals of the quantities introduced in (2), (3), and (5) respectively. For each complete magnetization cycle, \( E_{H} \) is zero and \( E_{ot}=W_{PC} \) is proportional to the area enclosed within its magnetization loop (see (5) and Fig. 3).

The energy lost in identical powering cycles was calculated with a precision of 0.5\%. The very high reproducibility of this loss is mainly due to the high precision of the power converter and of the DCCT.

**FIG. 4 HERE**

**IV. CONCLUSION**

A fast method to quantify the magnetization effect in an entire magnet was developed at CERN based on the
measurement of the voltage across the magnet and its current during a powering cycle. The calculated magnetization represents the collective effect of the magnetization in the superconducting filaments of the magnet, and of the magnetization of the iron yoke. At low ramp-rate the magnetization in the conductor is mostly due to the hysteresis in the superconducting filaments of the magnet since the contributions due to inter-filament and inter-strand coupling losses are limited.

The collective magnetization is calculated by means of an energy balance between the work done by the power converter and the change of magnetic energy in the system.

The difference between the calculated collective magnetization and the magnetization in the superconducting strands simulated with the numerical program ROXIE gives an indication of the energy lost due to magnetization effects other than the hysteresis in the superconducting filaments.

The energy lost due to the magnetization effect was measured in independent cycles with a precision of 0.5%.

The proposed method shows good precision and very high reproducibility, is fast and fairly easy to apply, and can be performed on any magnet type. Good-precision DCCT and power converters are needed for obtaining such high precision.

This method can be used for checking the reproducibility of the magnetization effect among a large series of magnets, and thus detecting possible outliers. It is especially useful to assess the magnetization effect as a function of the current in a magnet without a time-consuming measurement of field error with rotating or fixed coils.

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REFERENCES


Fig. 1. Measured voltage across the magnet $U$ and current $I$, and calculated ramp-rate, $dU/dt$ (Cycles 1 and 2).

Fig. 2. Calculated differential inductance, $L_d$ (Cycles 1 and 2).
Fig. 3. Calculated collective magnetization, $M_{\text{col}}$ (Cycles 1 and 2), ROXIE-simulated average strand magnetization $M_{\text{sc,ROXIE}}$, their difference (in arbitrary units), and saturation curves $M_{\text{sat}}$ for the inner and outer strands.

Fig. 4. Calculated work done by the system, $W_{PC}$, magnetic energy due to a change of $H$, $E_{H}$, and magnetic energy due to a change of $M$, $E_{M}$ (Cycles 1 and 2).