

# A Review on Quantum Metrology Triangle Experiments

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**Abstract** – Quantum Metrology Triangle experiments combine the three quantum electrical effects that are relevant in metrology, and thus enable a fundamental consistency test on the validity of commonly assumed relations between fundamental constants of nature and the quantum electrical effects. This paper reviews the history, results and the present status of Quantum Metrology Triangle experiments and their perspectives. Possible implications for the knowledge on fundamental constants and the quantum electrical effects are discussed.

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**Keywords** - Charge, current, voltage, fundamental constants, phenomenological constants, measurements; Josephson voltage standard, quantization, quantum electrical standards; quantization test, quantum Hall effect, quantum Hall standard, quantum metrology, quantum metrology triangle, SI, single-electron devices, single-electron tunnelling, unit system, units

## I. INTRODUCTION

The modern SI, based on seven base units, up to now has been very successful in ensuring worldwide consistency and uniformity of measurements. However, with scientific progress over the past decades, certain disadvantages are now apparent, in particular in the definition of the kilogram as the unit of mass, but also in the definition of the electrical base unit ampere. The impending revision of the International System of units (SI) presents fundamental metrology with the most profound paradigm changes since the implementation of the SI in 1960 [1, 2].

In the present SI, the kilogram is the last base unit still being based on a manufactured object. Like any artefact, this metallic kilogram body is susceptible to changes over time. The electrical base unit within the SI, the ampere, is presently still defined in terms of mechanical units of mass, length and time via the laws of classical electromagnetism. This is to be improved for two main reasons: firstly, the SI ampere is vulnerable to drift and instability from the kilogram artefact, and secondly, the electro-mechanical experiments needed to realise the SI electrical units are extremely difficult and require decades of effort. Moreover, under its present definition the ampere cannot be realised with accuracy better than a few parts in  $10^7$ . This is not sufficient to meet the accuracy needs of routine electrical metrology, which requires one part  $10^7$  now and will require even better in the future.

For about 30 years now, the Josephson and the quantum Hall effects, related to the fundamental constants  $h$  and  $e$  via the Josephson constant  $K_J$  and the von Klitzing constant  $R_K$ , have proven their unexcelled precision and reproducibility of the order of 1 part in  $10^9$  and better (see, for instance, [3]). In 1990 the 18th General Conference on Weights and Measures adopted the so-called “conventional units” for voltage and resistance ( $V_{90}$  and  $\Omega_{90}$ ) in order to exploit these effects for metrology (i.e. for the reproduction of the electrical SI units volt and ohm), to benefit from the increased precision in electrical calibrations and measurements. Correspondingly, fixed values for the Josephson and the von Klitzing constants ( $K_{J-90}$  and  $R_{K-90}$ ) were defined. Since then, the Josephson voltage standard (JVS) and the quantum Hall resistance (QHR) standard have been used for these metrological purposes with great success [3, 4]. The conventional electrical units have achieved wide acceptance and are commonly used since then. However, their introduction came with the price of a dilemma, since  $V_{90}$  and  $\Omega_{90}$  are not consistent with the SI definitions of the volt and the ohm. Therefore, it is desirable to find a better, non-artefact-based definition of the kilogram, and a consequent definition of the ampere that can be realized in an easier and more precise way. This, together with the need to restore coherence to the SI system and enable practical unit realizations via direct traceability chains to invariants of nature, has driven recent efforts towards the re-definition of the SI units. Thanks to progress made at National Metrology Institutes (NMI) during the last decades, the newly proposed SI unit definitions can be entirely based on fundamental constants of nature and will consequently allow units realizations which are highly accurate and invariable over time [1, 2]. Of particular importance for electrical metrology are the new definitions of the kilogram, which will be connected to a fixed value of the Planck’s constant  $h$ , and of the ampere, which will be based on a fixed value of the elementary charge  $e$ . Consequently, the importance of the quantum electrical effects for the realisation and conservation of the units will be further strengthened.

A crucial point for the application JVS and QHR for the future realization of the SI volt and ohm is the assumption that the relations  $K_J = 2e/h$  and  $R_K = h/e^2$  are exact. Providing experimental support for this fundamental assumption is still an ongoing goal of modern fundamental metrology research, and its need has been repeatedly emphasised by the international Committee on Data for Science and Technology (CODATA) [5, 6]. Empirical information on possible corrections to the predicted fundamental relations can be provided by consistency tests involving the Josephson and the quantum Hall effect in combination with the single-electron transport effect, as in Quantum Metrology Triangle (QMT) experiments.

The purpose of this paper is to review the current status of QMT experiments, including developments since the publication of previous review papers on the topic [7, 8]. In particular it is an update of the recent review on the topic given by [9]. Special focus is laid on the QMT realisation represented by the so-called “*Electron Counting Capacitance Standard*” experiment. The special role of superconducting devices and circuits involved in these experiments, like superconducting quantum interference detectors (SQUIDs) and cryogenic current comparators (CCCs), is mentioned.

## II. PRINCIPLE AND IMPLICATIONS OF THE QMT

Since the mid-1980s rapid advances in electron-beam lithography techniques have allowed the fabrication of sub- $\mu\text{m}$ -sized metallic tunnel junction systems. This initiated the advent of Single-Electron Tunnelling (SET) devices and brought ideas for corresponding metrological experiments and applications [10]. In 1985 the first formulation of a QMT was formulated and published [11], and a combination of three quantum electrical effects was proposed in order to investigate possible corrections to the underlying fundamental quantum relations.

About 15 years later, the QMT was re-interpreted [12], considering the constituting relations for the quantum electrical standards (Josephson voltage, quantum Hall resistance, and SET current) given by

- i)  $U_J = n f_J / K_J$  for the voltage produced by a Josephson voltage standard (JVS) operated at a frequency  $f_J$  and on the  $n^{\text{th}}$  voltage step,
- ii)  $R_{\text{QHR}} = R_K / i$  for the resistance of a quantum Hall resistance (QHR) standard operated on the  $i^{\text{th}}$  resistance plateau, and
- iii)  $I_{\text{SET}} = Q_S f_{\text{SET}}$  for the current generated by an SET current standard device, driving charge quanta of value  $Q_S$  at a frequency  $f_{\text{SET}}$ .

The main point of this re-interpretation was the notion that  $K_J$ ,  $R_K$  and  $Q_S$  are understood as “phenomenological constants”. These have to be considered *empirical quantities* whose values have to be determined experimentally. In particular, no *a priori* relation of these constants to any fundamental constants of nature (like  $e$  and  $h$ ) must be assumed.

Combining the three quantum effects in an experiment exploiting Ohm’s law,  $U = R I$ , readily results in

$$K_J R_K Q_S = i n (f_J / f_{\text{SET}}). \quad (1)$$

This relation represents the general result of any QMT experiment. It tests the consistency of the quantum electrical effects by checking if the product of the phenomenological constants involved is equal to a product of integer quantum numbers and a ratio of two frequencies. It is important to note that

- equ. 1) compares *dimensionless* products, i.e. all implications arising from QMT results are independent of any particular unit system, and
- the right side of equ. 1) is usually known with negligible uncertainty since frequencies can be measured with very high accuracy by state-of-the-art methods.

A couple of standard theories for the quantum electrical effects relate  $K_J$ ,  $R_K$  and  $Q_S$  to  $e$  and  $h$  [13 - 17]. These theories agree that the relations are

- ib)  $K_J = 2e/h$  for the Josephson constant,
- iib)  $R_K = h/e^2$  for the von Klitzing constant, and
- iiib)  $Q_S = e$  for the charge quanta constituting the electrical current in SET devices.

However, some publications exist on possible quantum-electrodynamical corrections to the von Klitzing and the Josephson constant in strong magnetic fields [18, 19], but the predicted dependencies are very weak with relative corrections of the order of  $10^{-19}$ , negligible for practical electrical metrology.

Relation iiib) formulates a seemingly evident fact: namely, that the charge value carried by the charge quanta in solid-state devices is equal to the value of the free electron charge in vacuum. The existence of corrections to the electron charge by many-body in solid-state systems was first postulated in 1970 [20]. According to this work, the value of the charge quanta in solids is subjected to quantum-electrodynamic corrections, and the renormalized electron charge value in metals is higher than the vacuum value by a relative increase of the order  $10^{-10}$ . Soon after, however, several contradictions to this argument were published [16, 17]. Presently this fundamental question is still considered under discussion [7, 8], and possible corrections to  $e$  should not be ruled out *a priori*. Regardless of any theoretical arguments, empirical tests to verify the exactness of the relations ib) - iiib) at the highest possible confidence level are fundamentally important. In this context, an important implication regarding Watt balance experiments is pointed out: commonly the Watt balance

[21] is interpreted in the sense of providing a precise link between the mass and Planck's constant according to  $m \propto h$  [22]. This conclusion, however, only holds strictly under the assumption that the relations ib) and iib) are exact.

Corrections to the phenomenological constants are commonly parameterized as [12]

$$\text{ic) } K_J = (1 + \varepsilon_J) 2e/h,$$

$$\text{iic) } R_K = (1 + \varepsilon_K) h/e^2$$

$$\text{iiic) } Q_S = (1 + \varepsilon_S) e.$$

Combined with equ. 1) this leads in a first order approximation to the expression

$$K_J R_K Q_S / 2 \cong (1 + \varepsilon_J + \varepsilon_K + \varepsilon_S). \quad (2)$$

This means, if there are no corrections to any of the three involved quantum electrical effects (all epsilon corrections equal to zero), the QMT provides a consistency check by testing the relation  $1 = 1$ . QMT results can be thus generally be expressed as

$$1 = 1 + \Delta_{\text{QMT}} \pm u_{\text{QMT}} \quad , \quad (3)$$

where  $\Delta_{\text{QMT}}$  is the measured deviation from the expected  $1 = 1$  relation, and  $u_{\text{QMT}}$  is the relative standard uncertainty attributed to the result.

If  $\Delta_{\text{QMT}} > u_{\text{QMT}}$ , the result would imply that at least one of the involved quantum effects has a correction term; however, this would still not allow to identify the effect. If  $\Delta_{\text{QMT}} < u_{\text{QMT}}$ , the QMT is “closed”, which means that evidence against corrections to the quantum effects is provided on a confidence level of  $u_{\text{QMT}}$ . In this case, though, the possibility of a cancellation between epsilon correction terms cannot be ruled out [7, 12].

### III. KNOWLEDGE OF PHENOMENOLOGICAL CONSTANTS' VALUES

It is necessary to consider the present knowledge of the values of the phenomenological constants in order to assess the metrological impact boundaries of QMT experiments. Discussions on the QMT often have formulated the ambitious ultimate target to reach a relative standard uncertainty  $u_{\text{QMT}}$  of about one part in  $10^8$  (see for instance [12]), or they even state that such uncertainty level is necessary for significant metrological impact [23]. However, a conservative analysis based on recent CODATA results [6] implies that the metrological impact threshold for QMT experiments is significantly lower, namely at an uncertainty level of about few parts in  $10^7$  [7, 9].

The latest adjustment of the fundamental constants was performed in 2010 by CODATA, including an update on possible corrections to the phenomenological constants  $K_J$  and  $R_K$  [6, 24]. The results were derived by least-squares adjustments of the phenomenological constants based on input data from a wide variety of experiments [5, 25, 26]. In part these calculations were carried out without the *a priori* assumption that the relations  $K_J = 2e/h$  and  $R_K = h/e^2$  are exact. So-called “relaxed conditions” were considered by introducing adjustable correction factors  $\varepsilon_J$  and  $\varepsilon_K$  for the observational equations. Corresponding adjustments then provided a set of best values for the epsilon correction terms. According to the CODATA analysis from 2010 [6] the results were (with all stated uncertainties here being “standard uncertainties”):

- $\varepsilon_J = (15 \pm 49) \cdot 10^{-8}$
- $\varepsilon_K = (2.8 \pm 1.8) \cdot 10^{-8}$

i.e. there is no significant correction to the predicted value of the Josephson constant (von Klitzing constant) at a confidence level corresponding to a relative uncertainty of about five parts in  $10^7$  (two parts in  $10^8$ ).

Astonishingly, the correction factor to  $K_J$  has a relatively high uncertainty of about five parts in  $10^7$ . This seems surprising since the Josephson effect is considered one of the best understood quantum electrical phenomena. The reason for the relatively high uncertainty is due to a peculiarity that was already revealed in an earlier CODATA report [5]. Considering the fact that the value for  $\varepsilon_J$  was mainly determined by different types of observational equations and experimental input data, it was found that different “routes” for the adjustment calculations led to strongly discrepant results for  $\varepsilon_J$  (see [7, 9] for an extensive discussion). In order to obtain a result free of inconsistencies, consequently additional adjustments were performed with all sets of discrepant input data deleted [5, 6, 26]. The adjustment then gave  $\varepsilon_K = (2.4 \pm 1.8) \cdot 10^{-8}$  and  $\varepsilon_J = (2.4 \pm 7.2) \cdot 10^{-7}$  [5]. Comparison with the latest results of the CODATA analysis [6] shows that the uncertainty for the correction factor to the Josephson constant now has slightly decreased from seven to about five parts in  $10^7$ .

In 2008, a value for the correction factor  $\varepsilon_S$  was deduced by combining the results of a QMT experiment performed at the *National Institute of Standards and Technology* (NIST, USA) [27, 28] with results from Watt balance and calculable capacitor experiments (see [29] and references therein). This gave  $\varepsilon_S = (-9 \pm 92) \cdot 10^{-8}$ , i.e. no significance for a correction to the predicted value of the charge quanta transported by SET devices at a confidence level corresponding to a relative uncertainty of about nine parts in  $10^7$ . This uncertainty value is dominated by the relative total uncertainty of the QMT experiment [27].

In summary: the relative uncertainty for a correction to  $R_K$  is about two parts in  $10^8$ , for a correction to  $K_J$  it is five parts in  $10^7$ , and for a correction to  $Q_S$  it is nine parts in  $10^7$ . Consequently, the implications of experimental QMT results are assessed as follows [7, 9]:

- A QMT result with an uncertainty  $u_{\text{QMT}}$  between nine parts in  $10^7$  and five parts in  $10^7$  has to be interpreted primarily in terms of  $\varepsilon_S$ .
- An uncertainty in the range about five parts in  $10^7$  and about two parts in  $10^8$  would have impact on  $\varepsilon_S$  and  $\varepsilon_J$  together, keeping in mind that a QMT result cannot distinguish between them according to equ. 2).
- A result with  $u_{\text{QMT}} < \text{two parts in } 10^8$  would bear on the correction factors for all three quantum electrical effects.

This means that any QMT result with a relative total uncertainty at the level of about a few parts in  $10^7$  can provide relevant input to future adjustments of the phenomenological constants. Such a result would contribute to reinforce with an empirical approach the theoretical models existing for the electrical quantum effects and their foundation as the basis for the future SI.

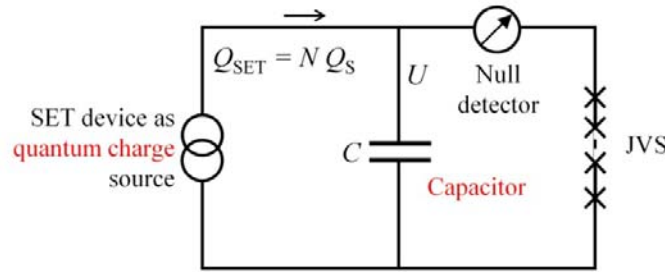
#### IV. DIFFERENT IMPLEMENTATIONS OF QMT EXPERIMENTS

At the time of the advent of the QMT idea [11], its experimental realization was not straightforward. When SET devices started entering metrology institutes, state of the art SET current sources were single-electron pump or turnstile devices based on series arrays of metal-insulator-metal tunnel junctions [10, 30]. Due to inherent physical limits set by the statistical tunnelling process, the quantized current levels achievable could not exceed the range of about few picoampere. A current of 10 pA driven through a quantum Hall standard operating on the highest resistance plateau ( $i = 1$ ,  $R_{\text{QHR}} = R_K \cong 25.8 \text{ k}\Omega$ ) results in a Hall voltage of about 40 nV. Measuring this voltage with a relative uncertainty  $< 10^{-6}$  requires an accuracy of

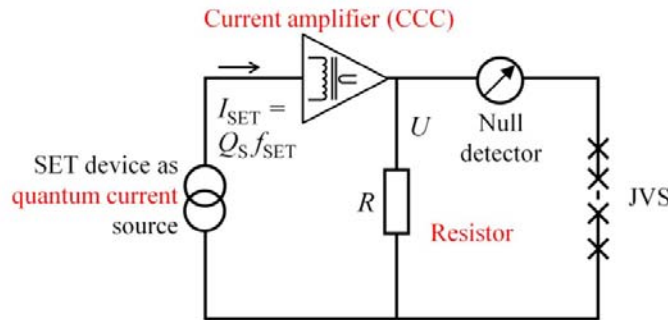
about  $4 \cdot 10^{-14}$  V, which exceeds the capabilities of JVS systems by far [4]. Therefore, a way to realize a QMT that avoided the practical difficulties arising from the limited SET current levels was needed.

In 1992, a proposal from NIST formulated a practically realizable QMT experiment [31] (Figure 1 a). The key idea is to accumulate the charge delivered by an SET pump on a cryogenic capacitor. For a suitably small capacitance value  $C$  (in the pF range) integration of the SET current over a period of a few tens of seconds would create a reasonably high voltage  $U$  - typically in the range of few volts - across the capacitor electrodes. This can easily be measured with a precision voltmeter, calibrated by means of a JVS. In this experimental scheme an SET electrometer was introduced to control the charging process of the capacitor and to verify the pump accuracy.

a) QMT in “charge” variant:



b) QMT in “current” variant:



**Fig. 1.** Basic principles of the two relevant variants of QMT experiments in schematic representation: a) shows the “charge” (ECCS, or indirect) QMT variant, and b) the “current” (Ohm’s law, or direct) variant (Figure from [9]).

In order to understand that this experiment is an equivalent representation of the QMT (according to equ. 1) we first consider the following rationale: the capacitance  $C$  of the capacitor can be traced to  $R_K$  via the QHR in the  $i^{\text{th}}$  quantized resistance plateau by using a suitable quadrature impedance bridge working at a frequency  $\omega$  according to

$$C = 1/(\omega R_{\text{QHR}}) = i /(\omega R_K) \quad (4)$$

With  $Q_{\text{SET}} = N Q_S$  (the total charge of  $N$  electrons moved between the capacitor electrodes by the SET device) and with  $Q_{\text{SET}} = C U$ , it follows

$$N Q_S = (U i) /(\omega R_K) \quad (6)$$

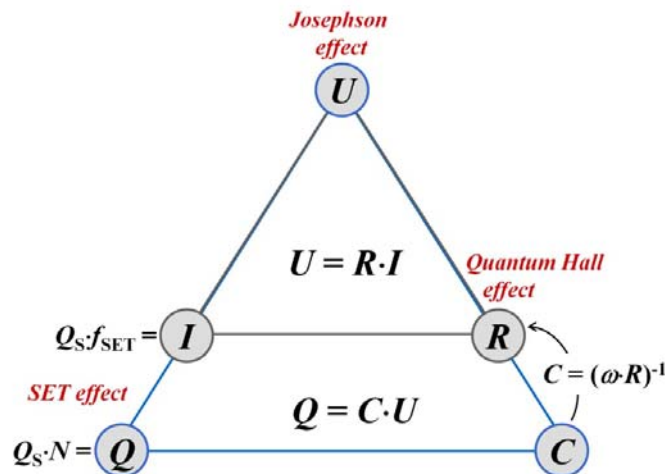
By measuring  $U$  using a JVS system (equ. i), we finally obtain  $N Q_S = (i n f_j / K_J) /(\omega R_K)$ , or

$$K_J R_K Q_S = (i n f_J)/(N \omega) \quad (7)$$

Hence, this experiment relates the product of the phenomenological constants to a product of integer quantum numbers and a ratio of two frequencies, analogous to equ. 1.

This experiment was first pursued by NIST and called “*Electron Counting Capacitance Standard*” (ECCS) [27, 32 - 34]. As the name indicates, in the beginning the experiment was meant to yield a new, quantum-based capacitance standard. Only about 8 years later it was interpreted in terms of a QMT experiment in “charge” version. Soon later, similar experimental setups were started at several European NMIs: the *Van Swinderen Laboratorium* (VSL, NL), the *National Physical Laboratory* (NPL, UK), the *Bundesamt für Metrologie* (METAS, CH), and the *Physikalisch-Technische Bundesanstalt* (PTB, D). This work was pursued in the frames of three joint European metrology research projects [35 - 37].

Another alternative to cope with the small SET currents in a QMT experiment was proposed in 2000 by the *Laboratoire National de Métrologie et d'Essais* (LNE, FR) [12]. This version of the QMT, schematically sketched in Figure 1 b), is based on high amplification of the SET current (by at least a factor of 10 000) by means of a CCC system in a dilution refrigerator environment. The amplified current is fed through a standard resistor, acting as a current-voltage converter traceable to the QHR. The resulting voltage is then directly measured by the use of a JVS system. The setup of this experiment was started at LNE and also pursued within the mentioned joint European projects [35 - 37]. Further progress in SET current source devices [38, 39] has later motivated the development of “direct” QMT versions which do not need the amplification of the SET current by high-gain CCC systems [40].



**Fig. 2.** Extension of the original version of the QMT (the “current” variant, cf. [11]), based on the Ohm’s relation  $U = R I$ , to the “charge” variant, based on the capacitance relation  $Q = C U$ . The link to the QHR is given by the impedance of the capacitance  $C$  (Figure from [9]).

QMT experiments based on the ECCS have already produced results at NIST [27, 28, and references therein] and later at PTB [41, 42, and references therein]. QMT implementations of “current” variant are still being developed by two European NMIs, which are LNE [12, 43, 44 and references therein] and *Mittateknikan KESkus* (MIKES, FI) [40, 45 and references therein]. The status and results of these experiments are discussed in more detail later in this paper.

Several new approaches to implement direct (or “current”) QMT variants are presently under development at some NMIs, among them NPL and PTB. They involve measurements of the SET current by advanced current-voltage conversion methods, for instance by using high-ohmic resistors traceable to the QHR [46]. However, to date this is still work in progress, and the experiments have not yet delivered significant results with respect to the QMT. Another, even more ambitious approach for a future QMT realization was recently proposed by PTB [47]. This idea is based on the monolithic integration of GaAs-based QHR and single-electron pump devices on a single chip.

In summary, two distinct variants are used for experimental implementations of the QMT, shown in Figure 1 and Figure 2. One is the “charge” or “indirect” QMT (based on the ECCS), which uses an SET device as a quantum charge source and needs a cryogenic capacitor. The other variant is the “current” or “direct” variant. It requires a suitable resistor and typically a CCC-based current amplifier with sufficiently high gain. Both versions include a null-detector instrument, whose performance typically limits the accuracy of these experiments: in the “charge” variant, an SET electrometer is used as a null-detector for voltage feedback [27, 42, and references herein] (see also Figure 6). The noise and drift of this electrometer, caused by movements of background charges around the SET device, set the accuracy limit of the experiment. In the “current” variant, the flux noise of the SQUID detector involved in the CCC amplifier setup typically sets the limits.

## V. NOTE ON ACCURACY VERIFICATION OF SET DEVICES

QMT experiments require proper operational conditions for the electrical quantum effects involved. For the QHR as well as for the JVS, well defined experimental criteria and methods are established in modern metrology. The exactness of the QHR and JVS benefits from the fact that the relevant experimental parameters are relatively easy to control in practical applications, as well as from the rather robust nature of the underlying macroscopic quantum effects. For instance, proper operation conditions here are assured by applying suitable bias currents and magnetic inductance, and by keeping the system at suitably low temperature; other relevant parameters are suitably low contact resistances of QHR devices, or suitably stable microwave frequency for irradiating JVS arrays.

It is important to note that this is generally more difficult for SET devices being used as current or charge standards. The preparation of their proper operation conditions is typically not straight-forward and complex, since it requires [48, 49 and references therein]:

- sub-Kelvin cryogenic environment by the use of dilution or He-3 refrigerator systems,
- thorough shielding of the SET devices from thermal background radiation, and
- extremely careful low-pass filtering of the experimental wiring to avoid electromagnetic rf interference effects (for typical SET current sources an attenuation of about 100 dB for frequencies of 1 GHz and above is required).

Special challenges arise not only because the nanometer-scale SET devices are electrically very fragile circuits that can easily be destroyed by handling during an experiment; but also because they are more susceptible to intrinsic error effects due to the underlying microscopic Coulomb blockade effects. For example in a “current” QMT experiment, the SET-generated current is described by the relation

$$I_{\text{SET}} = \langle n_{\text{SET}} \rangle \cdot Q_{\text{S}} \cdot f_{\text{SET}}, \quad (8)$$

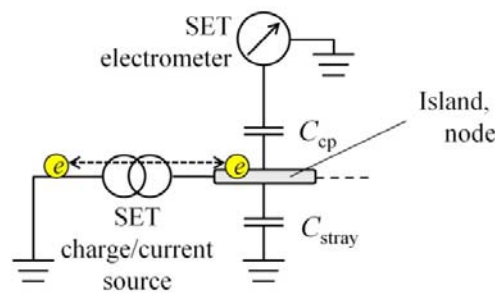
where  $f_{\text{SET}}$  is the driving frequency applied to the SET device and  $\langle n_{\text{SET}} \rangle$  is the average number of charge quanta (over many clock cycles) transferred per clock cycle with repetition



frequency  $f_{\text{SET}}$ . In the ideal case,  $\langle n_{\text{SET}} \rangle = 1$  for normal metallic and semiconductor SET devices, or  $\langle n_{\text{SET}} \rangle = 2$  for superconducting devices which pump Cooper pairs. In a real experiment, however,  $\langle n_{\text{SET}} \rangle$  typically deviates from the ideal value due to error effects. Such errors occur randomly in time during operation, and can, for instance, be caused by co-tunneling or other parasitic effects, or by “missed cycle” events. Errors can also be triggered by rf background interference or by thermal activation [10, 30, 48, 49 and references therein]. Depending on the experimental setup and the setting of the SET device operating parameters, this can lead to deviations from the ideally quantized behavior amounting to parts in  $10^6$  or more [43, 44 and references therein]. Proper tuning of SET device requires in particular the adjustment of their working point via external control parameters, typically dc and ac voltage levels on gate electrodes of the SET device. The same complications arise when SET devices are used as quantum charge sources in ECCS experiments [33, 34, 48, 50 and references therein].

In light of this, it follows that the quantitative accuracy verification of the single-electron transfer is an indispensable prerequisite for the metrological application of SET devices. Consequently for any QMT experiment, by definition aiming at a consistency check of the phenomenological constants, this verification must be done *independently* from SET current or charge measurements. An interesting finding from the LNE experiments was that deviations of  $I_{\text{SET}}$  from the expected quantized value  $e \cdot f_{\text{SET}}$  were observed although the measured current plateaux showed reasonable flatness [8, 44]. This strongly indicates that current plateaux flatness has to be considered a necessary, but not sufficient indication for the operation of SET devices as quantum standards.

In the ECCS experiment at NIST as well as in the PTB experiment, a quantitative determination of SET error effects is carried out by performing a preliminary “shuttle pumping” experiment (Figure 3): the SET device is connected to an on-chip metallic island provided with a small stray capacitance  $C_{\text{stray}}$ . This node is electrostatically connected to the input of an SET electrometer via a coupling capacitance  $C_{\text{cp}}$ , which provides sub- $e$  charge resolution as long as the ratio  $C_{\text{cp}}/C_{\text{stray}}$  is made sufficiently large by a suitable device design. The SET pump is operated so that it repeatedly pumps one electron in and out from the island, while the electrometer is used to monitor the charge state of the island.



**Fig. 3.** Schematic operational principle to detect transfer errors in an SET pump device by “shuttle pumping”. Single electrons are shuttled in and out from the island at the frequency  $f_{\text{SET}}$ . If the effective charge divider ratio  $C_{\text{cp}}/C_{\text{stray}}$  is sufficiently large, single electron charges on the island can be resolved by the SET electrometer, and pump errors can be detected (Figure from [9]).

The accuracy of the SET pump is determined by measuring the average rate of the error events detected by the electrometer and relating this number to the pumping frequency  $f_{\text{SET}}$ . The best results for SET pumps based on metallic tunnel junctions, obtained for frequencies

$f_{\text{SET}}$  of few MHz, correspond to relative single-electron transfer errors of about few parts in  $10^8$  [27, 28, 33, 51]. In the PTB experiment, values of better than one part in  $10^7$  are achieved routinely.

Although this procedure is suitable to quantify SET error contributions for uncertainty assessments, it has flaws. The main one is given by the assumption that the error rates during the shuttling phase, i.e. determined by bidirectional pumping of single electrons, are equal to the ones in the unidirectional pumping process phase of the experiment, when the SET current (or charge) is sourced to the resistor in a “current” QMT (or to a capacitor in a “charge” QMT). More advanced variants for SET error detection and accounting are currently pursued at PTB [52 - 54]. Here, the errors occurring in a serial array of (two or more) SET pump devices are detected on charge nodes between each two pump devices using SET electrometers as single-electron charge detectors. A logic circuit, processing the output signals of individual electrometers, allows the identification the error-producing device. Once errors are identified and quantified, they can be incorporated as known correction terms for the determination of the current or charge sourced by the device.

Reliable error detection requires SET electrometers with sufficiently large bandwidth. For very well-performing pump devices with error rates of the order of about  $100 \text{ s}^{-1}$  (corresponding, for example, to an accuracy of 1 part in  $10^6$  at a pumping frequency  $f_{\text{SET}} = 100 \text{ MHz}$ ), conventional dc-SET electrometers are still adequate. For less accurate devices with higher error rates, or for higher pumping frequencies, the detector must have a larger bandwidth. This is achievable by using rf-SET circuits operating with a typical carrier frequency of about 500 MHz and with a bandwidth around 1 MHz [55].

Several European NMIs, among them MIKES, NPL, and PTB, are currently pursuing the development of advanced SET error accounting schemes by means of monolithically integrated quantum current source/detector schemes. These activities are also supported by the Joint Research Project “Qu-Ampere” within the European Metrology Research programme [56].

## VI. RESULTS AND STATUS OF QMT EXPERIMENTS WORLDWIDE

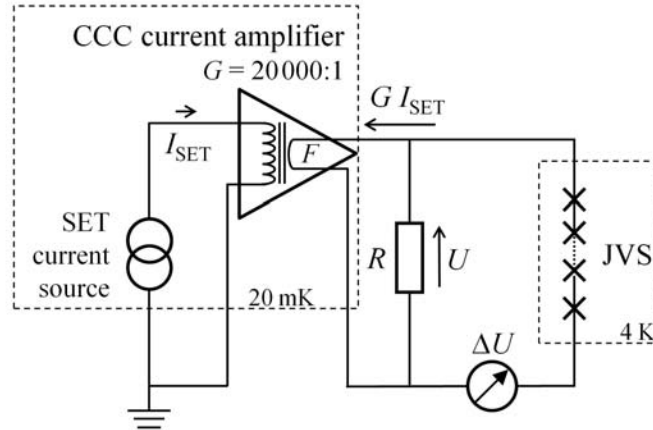
### A. The LNE “Direct” QMT Experiment

The QMT experiment at LNE, described in [8, 11, 42, 43, 57, 58], uses a 3-junction  $R$ -pump, developed and fabricated at PTB. A simplified scheme of the experimental setup is shown in Figure 4.

The  $R$ -pump is an improved concept of the conventional SET pump based on Al-AlO<sub>x</sub>-Al tunnel junctions [59]. It is equipped with on-chip chromium micro-strip resistors in series with the junctions, each resistor having a resistance exceeding  $R_K$ . The resulting modification of the effective electromagnetic environment of the junctions has been shown to suppress unwanted co-tunnelling events, which are presumed to compromise the accuracy of  $I_{\text{SET}}$ . At LNE the 3-junction  $R$ -pump was operated up at a maximum frequency  $f_{\text{SET}} = 100 \text{ MHz}$ , corresponding to  $I_{\text{SET}} = 16 \text{ pA}$ .

The specially developed current amplifier of LNE is composed of a CCC with a high winding ratio  $G = 20\,000:1$  together with a dc-SQUID, capable of amplifying  $I_{\text{SET}}$  to about  $0.3 \text{ }\mu\text{A}$ . A secondary current source is servo-controlled by the SQUID, which works as a null detector for the magnetic flux  $\Phi$  in the CCC. The polarity of the SET current to be amplified is periodically reversed in order to reduce contributions from  $1/f$  flicker noise. The voltage  $U$  across the room-temperature  $10 \text{ k}\Omega$  standard resistor is simultaneously measured by a

programmable JVS system in combination with a precision voltmeter. The irradiation frequency  $f_j$  of the JVS and also the pumping frequency  $f_{\text{SET}}$  are referred to a 10 MHz frequency standard.



**Fig. 4.** Principle of the QMT experiment at LNE, involving a CCC-based current amplifier and an  $R$ -pump as SET quantum current source. Two detectors were used: a SQUID (not shown) which detects and nulls the magnetic flux  $\Phi$  induced in the CCC, and a voltmeter to measure the deviation  $\Delta U$  from the quantized voltage given by the JVS system (Figure from [9]).

The preliminary results of the LNE experiment, presented in [8, 43, 57], showed a relative deviation  $\Delta_{\text{QMT}}$  from the expected QMT relation (equ. 3) of few parts in  $10^4$ , with a relative uncertainty of  $u_{\text{QMT}}$  of few parts in  $10^6$ . Considering the fact that the experiment suffered from irreproducibility problems observed in a series of measurements, and given the present knowledge on the maximum  $\Delta_{\text{QMT}}$  value to be expected, which is less than one part in  $10^6$  [7], those preliminary results hinted to problems of the experiment. However, recently improvements of the setup remedied the lack of reproducibility, and the best result achieved in the LNE experiment to date is  $Q_S/e - 1 = (-5 \pm 13) \cdot 10^{-6}$  [58].

The uncertainty for the LNE experiment is in principle limited by statistical uncertainty contributions dominated by the noise of the SQUID null detector. These uncertainty contributions are inversely proportional to the current and inversely proportional to the square root of the measurement time. The largest uncertainties related to systematic effects (type B components) are estimated to be on the order of one part in  $10^8$  or less, and depend weakly on the current level [37]. They arise from the CCC ( $u_{\text{CCC}} \cong 10^{-8}$  including capacitive leakage, finite open loop gain and winding ratio error), the calibration of the 10 k $\Omega$  standard resistor against a QHR (typically  $u_{\text{QHR}} < 10^{-8}$ ), and the systematic uncertainties related to the JVS system ( $u_{\text{JVS}} < 10^{-8}$ , mainly due to residual thermal voltages, resistive leakage, and detector and frequency errors). However, the LNE experiment lacks the means for an independent determination of the SET transfer errors (e.g. by shuttle pumping measurements), since it does not include a single-electron charge detector. Thus, uncertainty contributions related to the SET pumping errors cannot be quantified.

The ultimate accuracy limit of the experiment, assessed in frame of the REUNIAM project [37], is crucially dependent on the performance of the CCC including the SQUID. For  $I_{\text{SET}} = 1$  pA and a CCC input current resolution of 1 fA/ $\sqrt{\text{Hz}}$  it was estimated that a standard uncertainty of about four parts in  $10^6$  should be realistically achievable during a measurement time of 10 hours. Considering a relative standard uncertainty of one part in  $10^8$  as the ultimate, ambitious target for QMT experiments, it was further concluded that the LNE

experiment could be performed with such uncertainty if the following conditions were fulfilled:

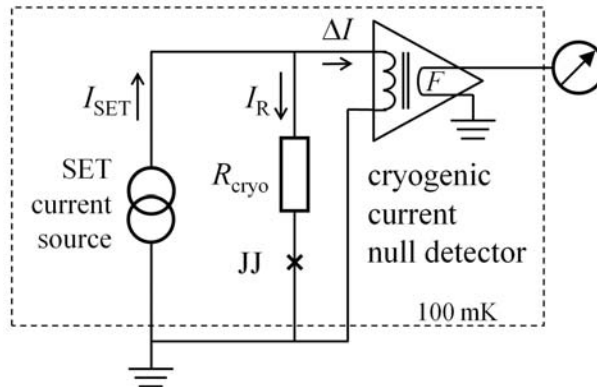
- availability of a CCC with a current resolution of  $1 \text{ fA/Hz}^{1/2}$  or less in the white noise regime,
- protection of the electrical wiring between the CCC and the SET current source against microphonic and interference pick-up effects, and
- availability of an SET current source generating  $I_{\text{SET}} \geq 100 \text{ pA}$  with highly stable performance.

### B. The MIKES “Direct” QMT Experiment

The QMT experiment under development at MIKES involves a hybrid turnstile device as SET current source, a cryogenic resistor, and a cryogenic current null detector. The experimental scheme described in [40, 45] is sketched in Figure 5.

Hybrid turnstiles are a new kind of SET quantum current source devices [39]. They comprise two metallic nano-scale superconductor–insulator–normal tunnel junctions in series. The interplay of the Coulomb blockade and the superconducting energy gap enables the clocked transfer of single electrons by using only one driving gate signal. These devices are categorized as “turnstile” since they must be operated with a finite bias voltage applied to their source-drain terminals, in contrast to pumps which are able to clock-transfer electrons without such bias [10].

The current  $I_{\text{SET}}$  delivered by the turnstile device is directly opposed to a current  $I_{\text{R}}$ , which is generated by applying a Josephson voltage to a cryogenic resistor with a resistance of  $1 \text{ M}\Omega$ . The small unbalanced current difference  $\Delta I = I_{\text{SET}} - I_{\text{R}}$  is detected by a cryogenic null detector, realized by a dc current transformer with moderate gain, in combination with SQUID as current null detector.



**Fig. 5.** Principle of the QMT experiment at MIKES. The current  $I_{\text{SET}}$  is balanced by an opposite current  $I_{\text{R}}$  from the voltage of a Josephson junction biasing a cryogenic resistor. The remaining difference  $\Delta I$  is measured by a cryogenic null detector including a dc current transformer and a SQUID, not shown here (Figure from [9]).

For the first experiments it is planned to operate the cryogenic resistor at a temperature of  $0.7 \text{ K}$ , which gives a Nyquist current noise of about  $6 \text{ fA}/\sqrt{\text{Hz}}$  [37]. The dominant statistical uncertainty contribution in this is, however, given by the noise level of the SQUID null

detector, generating a noise equivalent to 20 fA/ $\sqrt{\text{Hz}}$  or higher. In a later development stage, this is planned to be improved by using a null detector specially designed for this purpose. Assuming that the SET current device is generating a current of 100 pA at sufficient accuracy, the noise figures of the setup would limit the total relative uncertainty to about eight parts in  $10^7$ , requiring an averaging time of about 10 hours. These preliminary estimates have neglected possible flicker noise and drift effects which may appear when measurements are averaged over a very long time.

A preliminary assessment of the possible ultimate accuracy limit of this experiment shows that - besides the noise of the current null detector - the current dependence and possible flicker noise of the thin-film cryogenic resistor are the dominant type-A contributions [37]. The calibration of this resistor against a QHR at a current of about 1  $\mu\text{A}$  is possible with a relative uncertainty  $< 10^{-7}$  if a CCC bridge is used, but difficulties may arise since the maximum current of the SET device is limited to about 100 pA. This mismatch in current together with the current coefficient of the cryo-resistor may cause relative uncertainties of the order of few parts in  $10^6$  [60].

A significant improvement of the uncertainty of this QMT experiment below 1 part in  $10^6$  would require a better understanding of these current dependent effects and the availability of a null detector with lower noise floor. In addition it would need a drastic increase of the output current of the quantum current source by about a factor of 10 to reach the 1 nA level, which seems not possible at present but may be feasible in future, e.g. by a parallel combination of SET current source devices.

### C. The “Indirect” QMT Experiment (ECCS) at NIST

After the proposal for the Electron Counting Capacitance Standard in 1992 [31], NIST had continuously developed the corresponding experiment. In the beginning, the work focussed on the development of a suitable SET pump, starting with metallic single-electron pump containing five junctions in series [32]. Since the pumping accuracy was found to be insufficient, pumps with an increased number of junctions were developed in the following years. In 1996, the first 7-junction pump with sufficient pumping accuracy, i.e. with a relative uncertainty of only about 1.5 parts in  $10^8$ , was presented [33, 34]. Such pump was used in the first proof-of-principle demonstration of the ECCS [27].

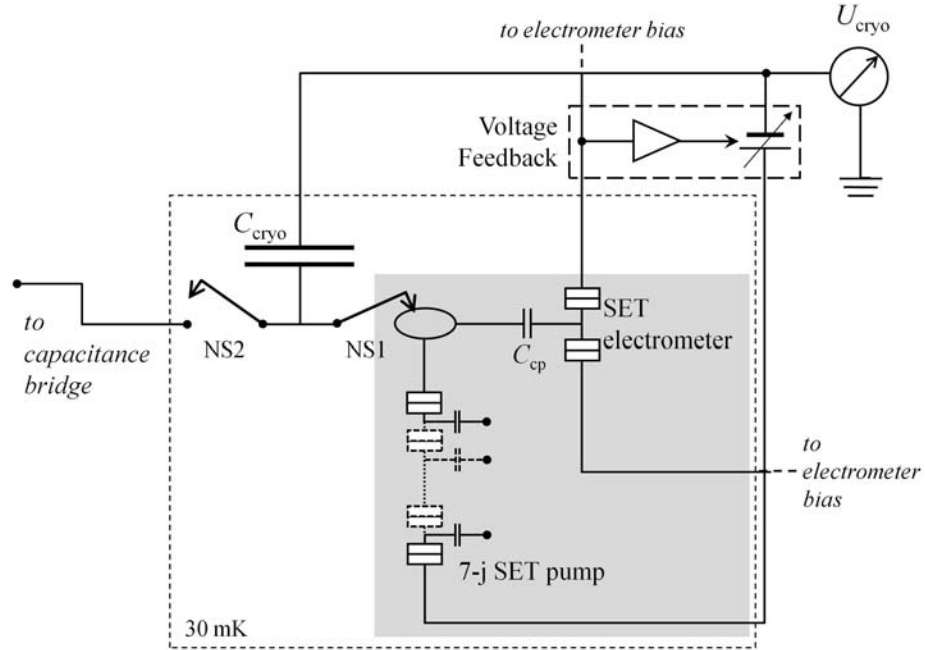
Besides the 7-junction SET pump combined with an SET electrometer on-chip, the NIST experiment comprised a vacuum-gap cryogenic capacitor ( $C_{\text{cryo}} \cong 2$  pF, in the following for simplicity called “capacitor”) with parallel-plate arrangement of the electrodes. Furthermore, two specially designed mechanical needle switches were used to provide switchable electrical contacts between the SET chip and the capacitor, or, respectively, between the capacitor and a capacitance bridge for measuring  $C_{\text{cryo}}$ . The experimental setup is schematically shown in Figure 6 and in detail described in [27, 28, and references therein]. Details on the experimental procedures as well as on the data analysis are also given in [27, 28].

The operation of the first prototype ECCS experiment [27] showed a reproducibility of few parts in  $10^{-7}$ , but lacked a full uncertainty analysis. The completion of the uncertainty budget required quantifying several systematic uncertainties, particularly the frequency dependence of the capacitor, which was accomplished in 2006 [61]. The uncertainty budget for the experiment (nicknamed ECCS-1) was then published in 2007 [28], and the result was

$$(\Delta_{\text{QMT}} \pm u_{\text{QMT}})_{\text{ECCS-1}} = (-0.10 \pm 0.92) \cdot 10^{-6}. \quad (9)$$

Since  $\Delta_{\text{QMT}} < u_{\text{QMT}}$ , ECCS-1 had “closed” the QMT with a relative uncertainty of about  $0.9 \cdot 10^{-6}$ . This was the first result of a QMT experiment, and is still the best result to date. In

the following years, the development of an improved experimental setup was pursued at NIST, and extensive practical knowledge on the operation of the ECCS was gathered [48]. However, due to technical problems with the fabrication of suitably accurate SET pumps, the successful implementation and execution of an improved ECCS experiment was not completed, and NIST stopped work on the ECCS in 2008.



**Fig. 6.** Scheme of the NIST ECCS experiment. The SET circuit (7-junction SET pump plus SET electrometer) is shown shaded. In the “capacitor charging phase” (shown here) the needle switch NS1 is closed to connect the SET pump to the capacitor. The SET electrometer acts as voltage null detector, controlling the servo voltage  $U_{\text{cryo}}$  driven by the feedback circuit. In the next phase, needle switch NS1 is opened and NS2 is closed. This allows connecting the capacitor to a capacitance bridge for measuring  $C_{\text{cryo}}$  (Figure from [9]).

Implications from a combination of the ECCS-1 experiment result (equ. 9) with those of a Watt balance experiment was discussed in [29]. This combination forms a QMT that yields a value for  $Q_S$  in terms of the SI coulomb, independent of the Josephson and quantum Hall effects. The result was

$$Q_S/e - 1 = (-0.09 \pm 0.92) \cdot 10^{-6}, \quad (10)$$

with an uncertainty given by the ECCS-1 experiment.

In summary, the best knowledge to-date about the QMT is represented by the ECCS-1 from NIST, implying that the validity of the relation  $R_K \cdot K_J \cdot Q_S = 2$  is experimentally proven with an uncertainty of about nine parts in  $10^7$ . Furthermore, it allowed deriving the value of the correction parameter  $\varepsilon_S$  for the SET charge quantum which was consistent with zero at the same uncertainty level [29].

#### D. The “Indirect” QMT Experiment (ECCS) at PTB

The ECCS experiment being pursued by PTB is similar to the NIST experiment, however it differs in significant points (see [41, 42, 51] and references therein):

i) The SET quantum charge device is an  $R$ -pump [59]. However, instead of a 3-junction device as used in the direct QMT experiment at LNE, the ECCS at PTB uses a 5-junction  $R$ -pump which has shown relative single-electron transfer errors corresponding down to few parts in  $10^8$  in shuttle-pumping characterization measurements [42, 51]. Given the fact that this pump only needs four gate electrodes to be tuned for adjusting the working point, the practical benefit is its easiness in use (compared to a 7-junction pump from NIST) without sacrificing too much performance in pumping accuracy [42, 51].

ii) The cryogenic vacuum gap capacitor used in the PTB experiment has a coaxial electrode arrangement with a capacitance  $C_{\text{cryo}} = 1$  pF [62]. Trimming of the capacitor electrodes allowed tuning  $C_{\text{cryo}}$  to the decadic value of 1 pF within  $10^{-5}$  (relative deviation). The robustness of the coaxial construction gives a reproducibility of  $C_{\text{cryo}}$  of about  $10^{-5}$  between thermal cycles, which enables high-precision capacitance measurements by the use of special bridge techniques [41, 42, 63]. Furthermore, the larger distance between the capacitor electrodes (5 mm for the PTB design vs. 50  $\mu\text{m}$  for the NIST design) makes the frequency dependence of  $C_{\text{cryo}}$  smaller than two parts in  $10^8$  [41].

iii) A high-precision capacitance bridge technique, developed at PTB [63], allows  $C_{\text{cryo}}$  to be measured in terms of  $R_K$  with an accuracy of few parts in  $10^8$ . Thus, the dominant uncertainty contribution in the ECCS-1 uncertainty budget is expected to be negligible in the final PTB experiment [28, 42].

After a significant improvement of the earlier SET chip design, first preliminary results of the ECCS experiment at PTB were published in 2012 [42]. A full uncertainty budget was not presented here because several systematic uncertainty contributions were not yet quantified. However, with a conservative (over-)estimation of these contributions the following preliminary result (nicknamed ECCS-2) was derived [9]:

$$(\Delta_{\text{QMT}} \pm u_{\text{QMT}})_{\text{ECCS-2}} = (-0.31 \pm 1.66) \cdot 10^{-6} \quad (11)$$

Like the former result from NIST (ECCS-1), this result was also consistent with zero and, thus, “closing” the QMT, however with a still slightly higher relative uncertainty of about 1.7 parts in  $10^6$ .

Since the publication of [42] the experimental setup at PTB was completed and extensive measurements have been performed. Significant progresses in the improvement of the pumping accuracy and in the JVS-based voltage measurement of  $U_{\text{cryo}}$  were achieved [51]. The results of the latest experiments allow the further quantification of uncertainty contributions:

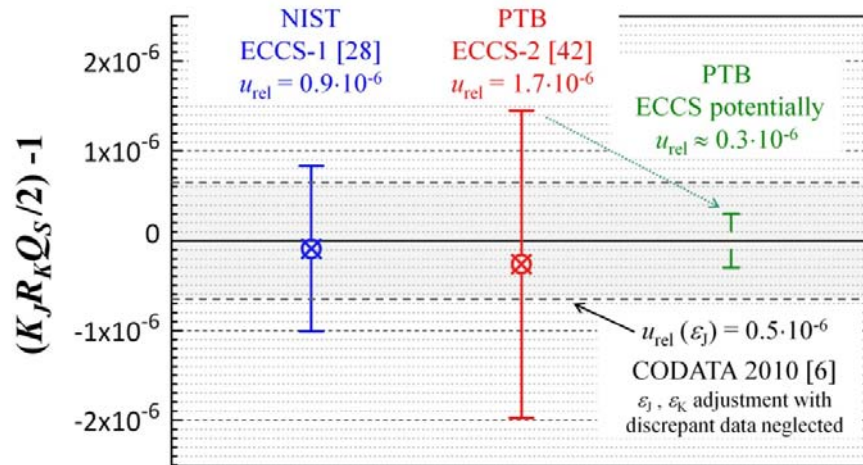
- The relative uncertainty due to pump errors achieved routinely is better than one part in  $10^7$ .
- The total relative statistical uncertainties from the capacitor charging phase of the ECCS can be brought to below few parts in  $10^7$  by repeated measurements.
- An upper limit of the systematic uncertainty contribution from possible leakage in the cryogenic capacitor was quantified to be less than one part in  $10^8$ . This corresponds to an insulation resistance higher than  $1.5 \cdot 10^{21} \Omega$  (i.e. charge leakage  $< 0.02 e/s$ ).
- The systematic uncertainty contribution from the voltage dependence of  $C_{\text{cryo}}$  was quantified to be less than five parts in  $10^8$ .

Also, high-precision capacitance measurements on the cryogenic capacitor including investigations of its frequency dependence have been performed using the technique described in [63]. These measurements unexpectedly showed results which require further investigations and probably further improvements of the setup. Once all further improvements

are implemented, the ECCS experiment at PTB is expected to produce results with an uncertainty level of down to three parts in  $10^7$ .

## VII. PRESENT STATUS OF THE QMT

A summary the results from the QMT experiments at NIST (ECCS-1 [28, 29]) and PTB (ECCS-2, [42]) in terms of their uncertainties is given in Figure 7, together with the uncertainty value for  $\varepsilon_j$  from the 2010 CODATA adjustment. The best result of the QMT experiment at LNE [58] is not shown here because it is not within the scale of the graph.



**Fig.7.** Results of the QMT experiments at NIST (ECCS-1) and PTB (ECCS-2) in terms of uncertainties for  $(K_J R_K Q_S / 2) - 1$ . The 2010 CODATA value for the uncertainty of  $\varepsilon_j$  ( $= 0.49 \cdot 10^{-6}$  [6]) is shown as grey horizontal line. The expected ultimate uncertainty limit for the QMT closed by the ECCS experiment at PTB is shown as green error bar.

A conservative interpretation of the latest CODATA adjustment result under “relaxed conditions” according to ic) – iiic) implies that a relevant QMT “impact” threshold regarding the uncertainty of  $\varepsilon_j$  is about five parts in  $10^7$  [9]. Up to date, no QMT experiment has reached this threshold. The ECCS experiment at PTB with an anticipated uncertainty limit of about three parts in  $10^7$  (green error bar in Figure 7), however, has the potential of reaching this impact threshold.

## VIII. CONCLUSION AND OUTLOOK

Developments of different QMT experiments over more than two decades, pursued at several NMIs worldwide, have shown that their setups require overcoming manifold difficulties and challenges, and therefore long-term efforts. This is not only because of the challenges with the operation of SET devices at metrological accuracy level; it is also because of the very nature of QMT experiments, which require all three quantum electrical standards operating properly in combination. As discussed in this article, the total uncertainty of the QMT experiments pursued currently may be reduced down to a few parts in  $10^7$  with the present methods.



For the “direct” or “current” type QMT experiments, involving a high-gain CCC or a cryogenic null detector, respectively, the most important condition is the availability of robust and highly stable SET current sources. These devices must be capable of delivering SET currents exceeding 100 pA significantly. Other obstacles remain to be overcome, particularly the reduction of the white noise floor of the complete system, corresponding to a current noise level of down to 1 fA/ $\sqrt{\text{Hz}}$  or less.

The “indirect” or “charge” type QMT experiment at PTB, also known as ECCS, has the potential to reach a total uncertainty of three parts in  $10^7$  after the completion of further improvements in reach. A result at this level would bear on possible corrections to both the SET charge quantum  $Q_S$  and the Josephson constant  $K_J$ .

To date all pursued QMT experiments still seem to cluster near an uncertainty level of about one part in  $10^6$ . However in the past years and ongoing, significant progress in the field of single-electron transport devices and their metrological application was made, in part described in a recent review paper [64]. In particular, significant progress towards GHz pumping rates and, correspondingly, higher pump currents was made [46, 56, 65-69]. In line with these developments, new advanced schemes for electron transfer error detection were recently proposed, and some already realized [52, 54, 56, 66]. All these advances go towards the realization of “self-referenced” quantum current standards, i.e. quantum current sources with integrated error counting detectors [56, 70]. These developments, in particular, might pave new ways for “current variant” QMT realizations. Ongoing exploration of further new concepts and material systems for quantum current sources, like those being based on graphene [67] or industry-level CMOS devices [68], can also contribute to further progress in the field. This supports the expectation that some QMT experiments in the near future may be capable of reaching an uncertainty level of few parts in  $10^7$ , and so of producing relevant impact for fundamental metrology. Closing the QMT at an uncertainty level of less than one part in  $10^7$ , however, remains a formidable task.

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