The Status and Future of Johnson Noise Thermometry

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Outline

1. Johnson noise thermometry
2. Absolute measurement
3. Different implementations and applications
4. Conclusion
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Johnson noise

- random thermal motion of electrons in a conductor causes both electrical resistance and a fluctuating voltage
- predicted by Einstein in 1906, measured by Johnson in 1927, and theoretically described by Nyquist in 1928
- fluctuation-dissipation theorem

\[ S_R = 4hfR \left[ \frac{1}{2} + \frac{1}{\exp(hf / kT) - 1} \right] \]  

\[ \langle V^2 \rangle = 4kT R \Delta f \]
Johnson noise thermometry

**Pros:**
- pure electronic measurement of thermodynamic temperature
- immune from chemical and mechanical changes in the material properties
- periodic calibration is not necessary

**Cons:**
- extremely small voltage, 100 ohm, 273K, \(\sim 1.2 \text{nV}/\sqrt{\text{Hz}}\) (amplify by \(10^5\))
- random, very long time integration \(\sigma \sim 1/\sqrt{t}\) (weeks or months)
- distributed over wide bandwidths \(\sigma \sim 1/\sqrt{\Delta f}\) (a few hundred kHz)
Switching correlator

- four wire connection defines the source impedance
- eliminates uncorrelated noise by cross-correlation
- eliminates the effect of amplifier gain drift by switching
- impossible to match both the noise power and frequency response
- affected by electronic nonlinearity or narrow bandwidth
- relative measurement, uncertainty limited to $10^{-5}$

\[
\frac{T}{T_{\text{ref}}} = \frac{\langle V^2(T) \rangle}{\langle V_{\text{ref}}^2 \rangle} / R
\]

H. Brixy, Nucl. Instrum. Methods, 97, 75-80 (1971)
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Primary thermometry

- primary gas thermometry limited by non-ideal properties of real gas
- JNT uses electron gas
- pure electronic approach, attracting increasing interest

Joachim Fischer, Metrologia 52 S364, 2015
Early attempt to measure $k_B$

To achieve 10 ppm in $k_B$ determination:

- 5 additional connections for calibration
- keep the temperature of all electronics constant within 0.02 K
- measure the divider factors with uncertainty less than 0.5 ppm
- accumulate data for more than 1 year!
Digital signal processing in frequency domain

\[ V_T(f) + V_{n1}(f) \]

\[ V_T^*(f) + V_{n2}^*(f) \]

Brixy introduced fast and accurate ADC to JNT

Digital signal processing in frequency domain

Bandwidth can be defined accurately

H. Brixy et. al., Temperature: It’s measurement and control in science and industry, vol 6, 993 (1992)
Quantum voltage noise source

Josephson Pulse Quantizer

Variable Input

Quantized Area $h/2e$

\[
\int V(t)dt = \frac{h}{2e}
\]

- Samuel Benz, Clark Hamilton (NIST)
- quantum accurate ($\ll 1$ ppm up to 4 MHz)
- calculable PSD
- arbitrary distribution

Quantum voltage calibrated noise thermometer

Johnson noise

Quantum voltage noise

\[ S_V \quad \rightarrow \quad K_J^2 = \frac{4e^2}{h^2} \quad \rightarrow \quad S_T \quad \rightarrow \quad R_K = \frac{h}{e^2} \]

\[ k = \left| \left\langle \frac{V_R^2}{V_Q^2} \right\rangle \right| \bigg|_{f=0} \quad \frac{\left\langle V_Q^2 \right\rangle_{\text{cal}}}{4TR} \]

John, Martinis (NIST)
Electronic measurement of $k_B$

- NIST reported first electronic measurement of $k_B$ with $u_r = 12.1 \times 10^{-6}$
- NIM/NIST collaboration, $u_r = 3.9 \times 10^{-6}$
- CCT required at least two methods with $u_r < 3 \times 10^{-6}$ to redefine the kelvin
- NIST, NIM, NMIJ, pursuing even lower uncertainty

CODATA 2010 $k_B$ input data

Mohr et al., Rev. Mod. Phys. 84 1527 (2012)
Benz et al., Metrologia 48 142 (2011)
QVNS-JNT system

\[ S_R = 4kT_W X_R R_K \]

\[ S_{Q\text{-calc}} = D^2 N_J^2 f_s M / K_J^2 \]

\[ k = \frac{D^2 N_J^2 f_s M}{4T_W X_R R_K K_J^2} \frac{\langle S_R \rangle}{\langle S_Q \rangle} \]

Qu et al., *Metrologia* 52 S242 (2015)
Shielding and grounding

- underground screened room
- shielding with aluminum and high-permeability nickle-alloy boxes
- powered by batteries
- eliminate ground loop

Measured spectra of the synthesized quantum noise waveform with (upper) and without (lower) observable EMI, blue green, and red are auto-correlation in each channel, and correlation spectra, respectively, and black × is the synthesized tones.
Effect of nonlinearity

- nonlinearity introduces significant errors
- PSDs are the same, Gaussian distribution, uncorrelated noise power are the same
- change the voltage of QVNS without changing any other parameters to measure the nonlinearity effect
- ~0. 4×10^{-6} error for 1% mismatch
Match the noise sources and transmission lines

- insert uncorrelated resistor to match both the noise powers and impedances
- insert trimming inductance and capacitance to match the transmission lines
Measurement result

Each measurement integrated for about 15-20 hours

45 measurements accumulated
Polynomial fit

- short connections-lumped components
- even-order polynomial fit
- increase bandwidth
- uncertainty increase with the number of fitting parameters
- ambiguity–which model to use?

\[
R(f) = \frac{S_R}{S_Q} \left( 1 + a_2 f^2 + a_4 f^4 + a_6 f^6 + \ldots \right)
\]
contour plot of total uncertainty versus model complexity and bandwidth

cross-validation method  (Kevin, Coakley et. al., arXiv:1606.05907)

select the optimal polynomial model and bandwidth by minimizing the uncertainty that accounts for both random and systematic effects
### Uncertainty Budget

\[
\frac{(k_B - k_B^{2010})}{k_B^{2010}} = +1.8 \times 10^{-6}
\]

<table>
<thead>
<tr>
<th>Component</th>
<th>( u_r / 10^{-6} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>3.2</td>
</tr>
<tr>
<td>Correction Model ambiguity</td>
<td>1.8</td>
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<tr>
<td>Dielectric losses</td>
<td>1.0</td>
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<tr>
<td>EMI</td>
<td>0.4</td>
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<tr>
<td>Nonlinearity</td>
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<tr>
<td>( R ) measurement</td>
<td>0.53</td>
</tr>
<tr>
<td>TPW</td>
<td>0.35</td>
</tr>
<tr>
<td>QVNS waveform</td>
<td>0.1</td>
</tr>
<tr>
<td>( u_r(k_B) )</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Qu et al., *Metrologia* 52 S242 (2015)
What’s next?

Problems to solve:
- drifting trend, probably caused by instability of the stray impedance
- the stray inductance dependent on both frequency and temperature
- thin, low TCR, coax cable
What’s next?

Problems to solve:
- match source impedance to the transmission line characteristic impedance
- four channel system to halve the measurement time (NIST)

Uncertainty limitation?
- fundamental limitation?
- practical limitation
  - bandwidth
  - time (1000 h for 3 ppm, 1 year for 1 ppm!)
What’s next?

High temperatures
- different fixed-point temperatures of Zn, Ag, Cu and Pd (PTB)
- uncertainty < 0.004% has been demonstrated up to 800 K (NIST)
- $T-T_{90}$, by NIST, NIM, NMIJ, PTB
- could be competitive in 600-1000 K with AGT and radiation thermometry

SQUID-based noise thermometer

Kamper, Zimmermann, JAP 42 (1971), 132

- R-SQUID noise thermometer


- Current Sensor Noise Thermometer


- Magnetic Field Fluctuation Thermometer

1 mK – 5 K, thermodynamic temperature
Tuned-RLC noise thermometer

\[ \text{ENBW} = \frac{1}{4RC} \]

\[ \langle V^2 \rangle = \frac{kT}{C} \]

Pepper, Brown, J Phys. E, 42 (1979), 31

- the noise amplitude is determined by a capacitor
- resistance measurement is not necessary
- inductively coupling allows non-contact measurement
- steel industry
Dual noise-thermocouple thermometer

- Combine noise and thermocouple thermometry
- Proposed by Brixy for use in nuclear plant or space satellite power system
- Calibration in situ
- Recently demonstrated up to 1450 °C at PTB
- Uncertainty of 0.1% under lab conditions, and double under industrial conditions

Dual noise-resistance thermometer

- single sensor, fast resistance mode or slow noise thermometry mode
- continuous AC signal is used to calibrate

from David Holcomb, ORNL
Practical noise thermometer

Inject precision current as calibration signal

- 5 kΩ sensor, 1MHz bandwidth
- Standard deviation 0.14°C at 20 °C

(from Paul Bramley of Metrosol)
Possible new applications

- remaining challenges: strong EMI, harsh environments
- rapid progress of electronics made it viable for industry

- suited for high-temperature, high accuracy applications:
  - next generation of nuclear power plant (~850 °C)
  - emission controls (0.1 °C @ ~850 °C)
  - aerospace (satellite with significant solar exposure, ionizing radiation)
  - high value manufacture (turbine, technical ceramics)
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Summary

- purely electronic approach, appealing alternative to other primary thermometry
- technology breakthroughs (switch correlator, ADC, QVNS) made it possible to contribute to the redefinition of kelvin
- could be competitive with AGT and radiation thermometry in range of 600 K - 1000 K
- different implementations have been demonstrated under lab conditions, cover temperature from millikelvin to over 1500 °C
- high temperature, high accuracy applications in industry becoming more practical
Acknowledgement

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- **NPL:** Jonathan Pearce

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Thanks for your attention!