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Effect of LC-shunting on the IV-characteristics of a Josephson Junction under Microwave Radiation.

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Abstract

We study the resonance features of the coupled system of Josephson junctions shunted by the LC-elements under electromagnetic irradiation. A strong effect of the external radiation on the IV-characteristics and voltage-time dependence is demonstrated. Crucial changes are found at the resonance condition when radiation frequency coincides with the Josephson and resonance circuit frequencies. It changes the amplitude dependence of the Shapiro step width. The optimized LC shunt leads to the increased step height for steps on the resonance branch of IV-characteristics at low amplitudes. The shunting of the Josephson junctions provides an extended range using the same microwave source, because Shapiro step demonstrates the first Bessel maximum at a much smaller power of radiation in comparison to the case of unshunted Josephson junctions. These features of Shapiro step on the resonance branch might be interesting for quantum metrology.

Models and method

Let us consider the system, presented in Fig. 1.

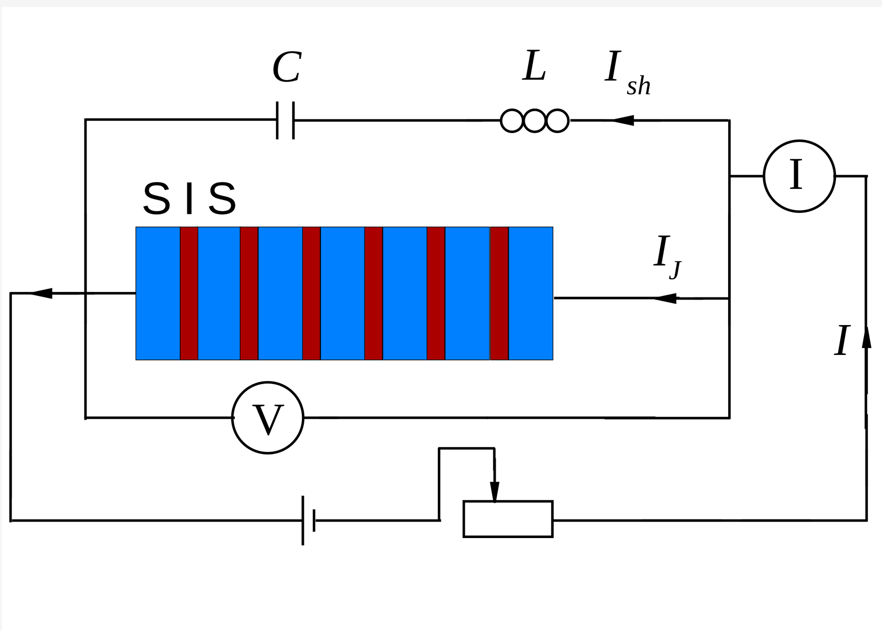

Fig.1 Stack of JJ shunted by LC-circuit

In normalized units the system of equations, describing this electric scheme can be written in the form [1, 2]

$$\begin{cases} \dfrac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l) \\ \dfrac{\partial V_l}{\partial t} = I + A\sin\omega_R t - \sin\varphi_l - \beta\dfrac{\partial \varphi_l}{\partial t} - C\dfrac{\partial u_c}{\partial t} \\ \dfrac{\partial^2 u_c}{\partial t^2} = \dfrac{1}{LC}\left(\displaystyle\sum_{l=1}^{N} V_l - u_c\right) \end{cases} \tag{1}$$

Here u_c is the voltage at the capacitance. The bias current I is normalized to the critical current I_c of JJ, A and ω_R are the amplitude and frequency of external electromagnetic radiation, time - to the inverse plasma frequency $\omega_p = \sqrt{\dfrac{2eI_c}{C_j\hbar}}$, voltages V_l and u_c are normalized to $V_0 = \dfrac{\hbar\omega_p}{2e}$; shunt capacitance C- to the capacitance C_j of the JJ, and shunt inductance L - to $(C_j\omega_p^2)^{-1}$. In the system of equations (1) we introduce a dissipation parameter $\beta = \dfrac{1}{R_j}\sqrt{\dfrac{\hbar}{2eI_cC_j}} = \dfrac{1}{\sqrt{\beta_c}}$ with β_c as McCumber parameter.

We study the phase dynamics of the system based on the eq.(1) and discuss the influence of LC shunting on IV-characteristics and resonance features in this system. We note that the JJs together with LC-elements form parallel and series resonance circuits with their eigenfrequencies

$$\omega_{rc} = \sqrt{\frac{1+NC}{LC}}, \quad \omega_{rc}^{s} = \sqrt{\frac{1}{LC}}, \tag{2}$$

where N is the number of JJs in the stack.

The width of SS in the absence of shunting is determined by [3]

$$\Delta I = 2|J_n(f)|, \quad f = \frac{A}{\omega_R}\frac{1}{\sqrt{\beta^2+\omega_R^2}} \tag{3}$$

where J_n is the Bessel function of the n–th order. The argument f depends on the frequency and amplitude of external radiation and the parameter of dissipation β.

In our calculations, the system of equations (1) is solved by the 4th order Runge-Kutta method.

Resonance Branch

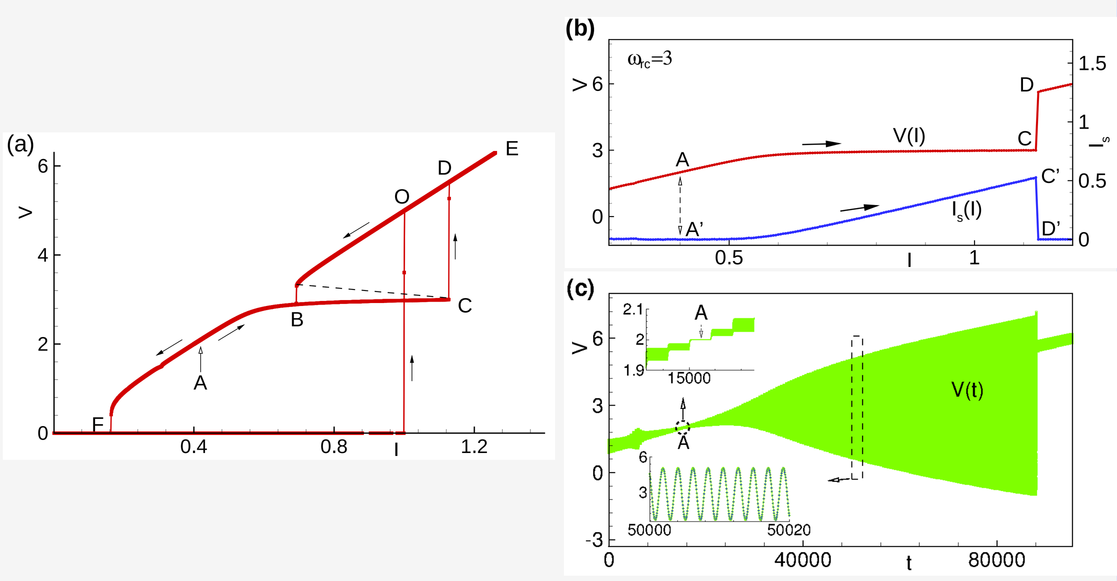

Fig.2 **(a)** The IVC of the shunted JJ, simulated by up and down sweeping of the bias current. The arrows indicate the direction of sweeping. There is a resonance branch between points A and C. Dotted line corresponds to unstable part of the resonance peak. **(b)** IVC and $I_s(I)$ of the JJ calculated with an increase in the bias current. The arrows indicate the direction of current sweeping. The characteristic points of $I_s(I)$ dependence are marked by capital letters with prime. **(c)** Time dependence of the JJ voltage which corresponds to the IVC. The top insert demonstrates a disappearance of Josephson oscillations at point A. A step like behavior of voltage on time is a numerical effect related to the jumps in bias current in the simulations. The bottom insert shows enlarge part of voltage oscillation. [4]

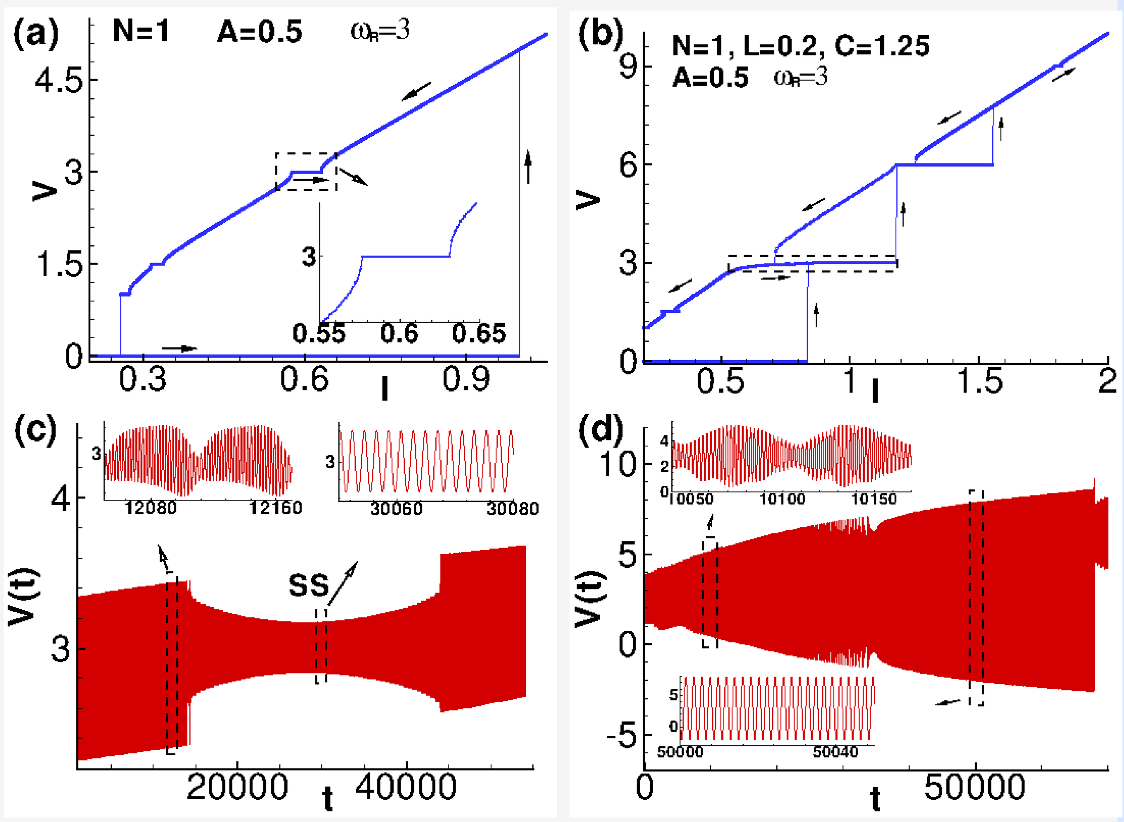

Fig.2 Time dependence of voltage and IV-characteristics of JJ under external radiation with frequency $\omega_R = 3$ and amplitude $A = 0.5$. **(a)** The IV-characteristics of JJ without shunting, inset enlarges the SS. **(c)** Time dependence of voltage for the labeled area on IV-characteristics. Inset enlarges oscillations of voltage which correspond to before the SS (left) and the SS (right) regions. **(b)** The IV-characteristics of JJ shunted by $L = 0.2$ and $C = 1.25$ elements. **(d)** Time dependence of voltage of rc-branch. Insets enlarge the oscillations of voltage which correspond to the regions before the SS (upper) and SS (down).

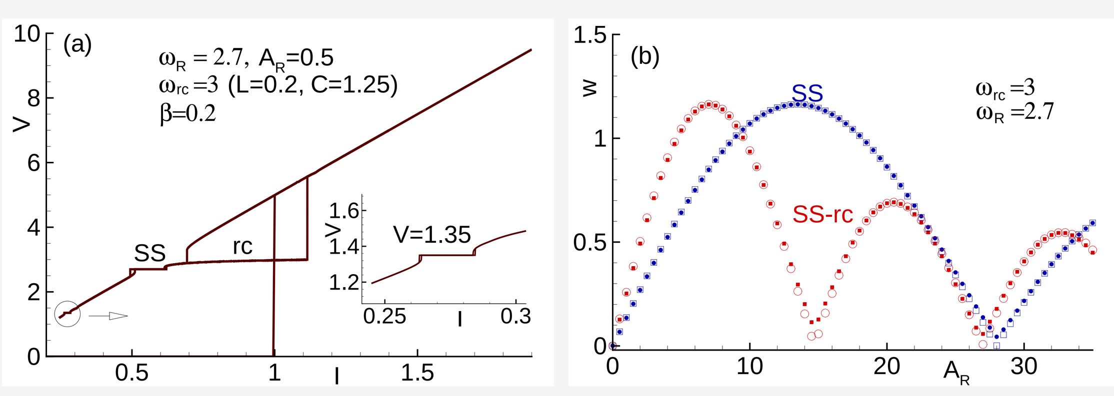

Fig.6 **(a)** Effect of radiation with the frequency $\omega_R = 2.7$ and amplitude $A = 0.5$ on the IVC of the shunted JJ. The inset enlarges the part marked by the circle demonstrating SS subharmonic; **(b)** The amplitude dependence of the width of SS on the resonance branch (SS-rc) at $\omega_R = 2.7$, compared to the dependence without shunting (SS). Blue (red) dots are the amplitude dependence of Shapiro step width without shunting (with shunting), calculated numerically. Blue squares correspond to the formula (3), red squares correspond to the formula (3) with changing $\omega_R = 2.7$ to an effective frequency $\omega_{ef} = 1.955$ as a fitting parameter. [4]

We see that the results of simulations in the case without shunting are in good agreement with theoretical equation (3). The results marked by the SS-rc show the dependence when SS is on the resonance branch. We see that the amplitude dependence of the Shapiro step width are crucially changed in the case of shunting. At small A the width is larger in the case of shunting than the width of SS for JJ without shunting. The period of the Bessel function is decreased in comparison with the case of the Josephson junction without shunting.

Influence of External Radiation

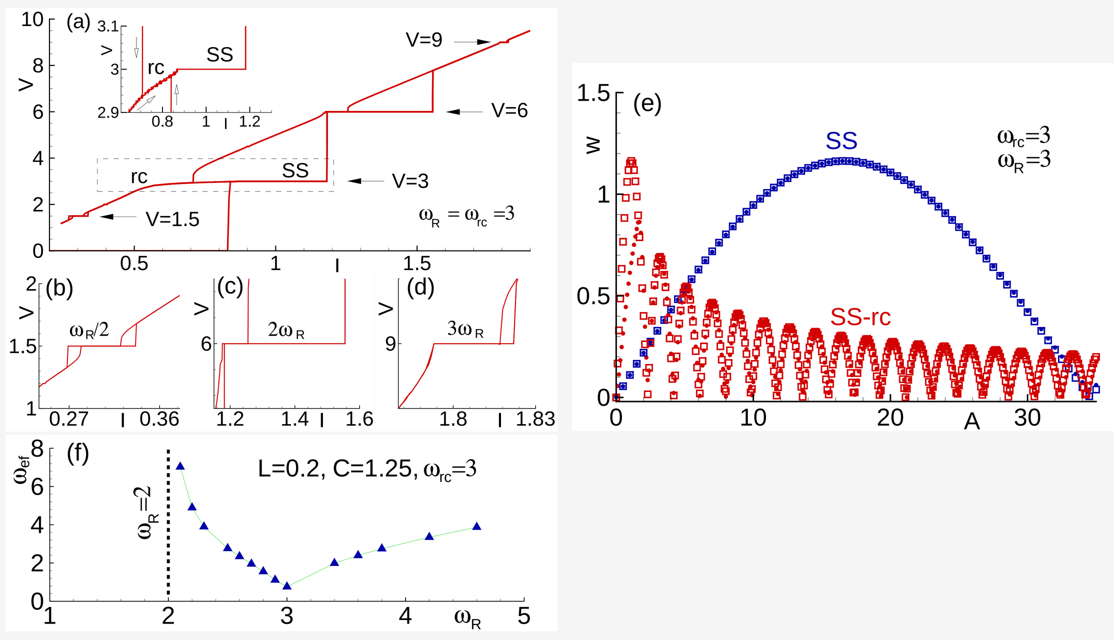

Fig.7 **(a)** The same as in Fig.6 (a) for $\omega_R = 3$. The arrows indicate the branch at the resonance $\omega_R = \omega_{rc} = 3$, harmonic at $V = 6$, $V = 9$ and subharmonic at $V = 1.5$. The inset enlarges the part marked by dashed rectangular demonstrating SS on the resonance branch. The arrows in the inset show the direction of bias current sweeping; **(b),c** and **(d)** enlarge harmonics and subharmonic marked in (a); **(e)** The same as in Fig.6(b) for $\omega_R = 3$ and $\omega_{ef} = 0.76$; **(f)** Dependence of the effective frequency on the radiation frequency at $\omega_{rc} = 3$. Dashed line stresses the dependence around onset of the resonance branch. [4]

To clarify changes of Bessel behaviour, we investigate the influence of the external radiation at different frequencies in the interval $2.0 \leq \omega_R \leq 4.6$ for $\omega_{rc} = 3$ and found the effective frequency ω_{ef} in each case. The results are demonstrated in Fig.7(f), where ω_{ef} is shown as a function of ω_R. We see a decrease of the effective frequency ω_{ef} until approaching the resonance condition $\omega_R = \omega_{rc}$, where the effective frequency has its minimum. Approaching the point A in the voltage scale by radiation frequency leads to the increase of the effective frequencies. The observed phenomenon can be manifested at different resonance circuit frequency. We get qualitatively the same result in case $\omega_{rc} = 4$.

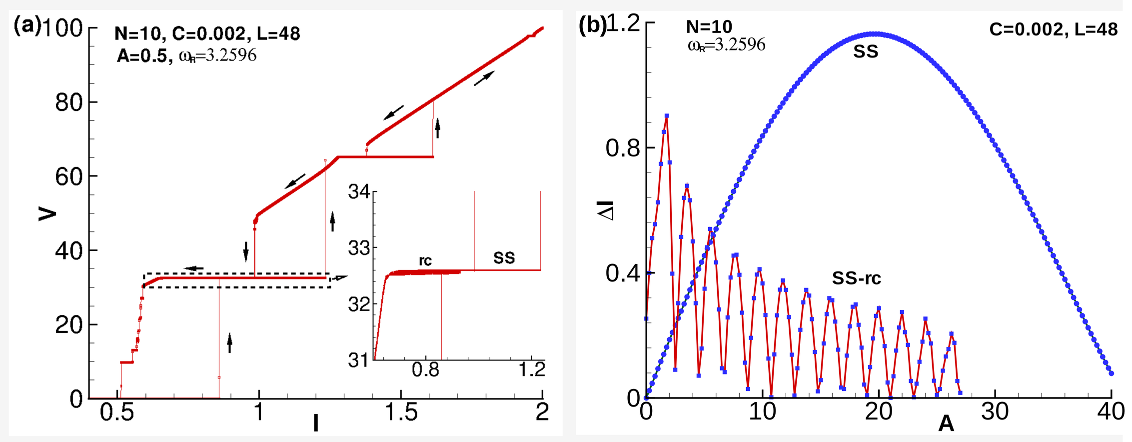

Fig.8 **(a)** The IV-characteristics of the stack with 10 JJ under external radiation with frequency $\omega_R = 3.2596$ and amplitude $A = 0.5$. Inset enlarges the rc-branch demonstrating Shapiro step on it. **(b)** The amplitude dependence of the first SS ΔI on the resonance branch (SS-rc) at $\omega_R = 3.2596$, compared to the same dependence without shunting (SS).

We see that the amplitude dependence of the Shapiro step width are crucially changed in the case of shunting. At small A, ΔI_{SS-rc} is larger in the case of shunting than ΔI_{SS} for the system of JJs without shunting. The period of the Bessel function is decreased in comparison with the case of JJs without shunting. We note that the amplitude dependence of ΔI in shunted case was calculated up to $A = 27$.

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