

Is SU(2) – a model linking superconductivity and charge density waves - the way to describe the basic physics of cuprate superconductors?

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Abstract— Charge density waves compete with superconductivity in cuprate materials, so it is important to understand them. A recent theory links charge density waves and superconductivity via an approximate SU(2) symmetry. We give a simplified version of this theory and discuss a few experiments which suggest that it may give a model that captures the essential physics of High- T_c cuprate superconductivity.

Index Terms— High-temperature superconductors, Quantum mechanics, Solid-state physics, Superconducting transition temperature, Yttrium barium copper oxide

I. INTRODUCTION

IN the last few years, it has become clear that charge density waves (CDWs) are an omnipresent phenomenon in slightly underdoped cuprate superconductors. These CDWs consist of slight modulations of doping and small displacements of atoms in the CuO₂ planes. They occur over similar temperature and energy ranges to superconductivity and compete with it. One may ask whether the CDWs represent yet another complication in the crowded phase diagram of these materials as a function of doping, or whether they give a deeper insight into the underlying phenomena. In either case, CDWs are worthy of attention because there are some indications [1] that if CDWs can be suppressed, the maximum T_c for superconductivity is increased. Here we give an extremely simplified account of a recent theory that links together the phenomena of superconductivity (SC) and CDW, and also gives an explanation of the mysterious pseudogap, which overlies the underdoped region of the phase diagram. A small selection of experiments that are consistent with this theory are called in evidence.

Manuscript receipt and acceptance dates will be inserted here. E. M. Forgan is supported by the The Leverhulme Foundation. EM-2015-029. (*Corresponding author: Ted Forgan.*)

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II. SU(2) THEORY

A. Superconductivity and Charge Density Waves

This theory [2, 3] calls attention to the fact that, unlike many other systems exhibiting CDWs and superconductivity, in the cuprates these two phenomena have a similar temperature scale. For example, in the transition metal dichalcogenides, superconductivity occurs at a temperature which is an order of magnitude below the onset of the CDWs. It has been clearly demonstrated [4] in underdoped YBCO that SC & CDW orderings compete with each other and therefore involve the same group of carriers. In addition, they can be described by similar theories. A CDW can be described as the Bose condensation of paired electrons and holes [4]; this does not lead to superfluidity, but if the electrons and holes have different momenta, it results in a spatial variation of charge density with a period corresponding to the difference of their \mathbf{k} -vectors. The critical temperature of the CDW is also given by a BCS-type expression in the weak coupling limit. Similarly, SC may be qualitatively described as the Bose condensation of pairs of electrons or pairs of holes, with T_c given in the weak coupling limit by the BCS expression. If the pairs of carriers have opposite momenta, then the superconducting condensate is at rest. If the sum of their momenta is nonzero, then this represents a supercurrent with all the pairs in the condensate having the same momentum.

B. The SU(2) model

SU(2) is the symmetry of quantum spin, but it should be emphasised that the present theory does not relate to actual electronic spin. Instead the behavior can be mapped to the SU(2) group, by defining an appropriate pseudospin, which is used to describe the relationship between closely related states. An analogue in particle physics is isotopic spin, which describes the proton and neutron as the two orientations of an $s = 1/2$ isotopic spin. As described in Ref. [2], various theories invoking symmetry have been proposed for cuprates, including some invoking SU(2), but the present approach appears to give an account of a much wider range of phenomena than earlier attempts. The present model for the cuprates is described in terms of a total pseudospin $s = 1$; the three related

states for this value of s may be visualized by reference to Fig. 1, and are described in terms of the z -components of their pseudospin as follows:

$s_z = +1$: Cooper pairing of electrons in $+\mathbf{k}$ and $-\mathbf{k}$;

$s_z = 0$: CDW pairing of electrons in $+\mathbf{k}$
 with holes in $+\mathbf{k}' = +\mathbf{k}+\mathbf{q}_{\text{CDW}}$;

$s_z = -1$: Cooper pairing of holes in $+\mathbf{k}'$ and $-\mathbf{k}'$.

It is the contention of the SU(2) theory that these three states should be treated on an almost equal footing (exactly equal footing if SU(2) symmetry is unbroken), and in particular that in the underdoped region the superconducting state should be described as a coherent mixture of these states. This has the consequence that the superconducting state should exhibit a “pair density wave”, i.e. a spatial modulation of the amplitude of the superconducting order parameter with the *same* repeat distance as the CDW. This modulation may be a small fraction of the total amplitude of the SC order parameter, which does not pass through zero at any point in space.

C. The role of antiferromagnetism in the model

It may be asked what role antiferromagnetic (AFM) interactions play in this model. This is illustrated by Fig. 2, where two AFM \mathbf{Q} -vectors are shown along with the antiferromagnetic Brillouin zone boundary. It will be seen that the carrier \mathbf{k} -vectors taking part in the pairing are at the “hot spots” connected by the AFM ordering vector to another part of the Fermi surface (FS). It will be noticed that both Cooper pairing and CDW connect states with the same sign of superconducting order parameter. However, as expected for pairing caused by a repulsive interaction, the AFM \mathbf{Q} -vectors connect states with opposite sign of the superconducting order parameter. In the full theory, all 8 hot spots are involved.

D. The “pseudogap” and “Fermi arcs”

In a region of the phase diagram where AFM ordering is removed by doping, the geometry shown in Figs. 1 & 2 is no longer exact. However, there will be AFM fluctuations available over an extended region of \mathbf{k} -space to enable superconducting pairing. Similarly, it is proposed that the SU(2) model applies in an extended region around the hot spots. Observations of the field-dependent part of the Nernst effect [6] suggest that there are fluctuations of incoherent pairing in the normal state above the SC T_c . However, these do not extend up in temperature throughout the whole of the pseudogap region of the phase diagram. Calculations using the model [2] give SU(2) fluctuations spreading around extended regions of FS close to the Brillouin Zone boundaries, and it is proposed that these exist throughout the pseudogap phase. The \mathbf{k} -space extent of these fluctuations depends on doping as shown in Fig. 3. This suggests that they cause gapping of increasing re-

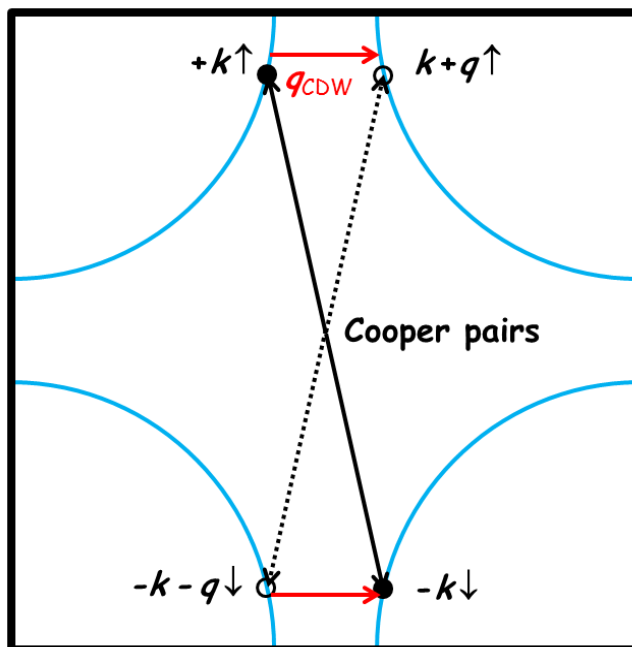


Fig. 1. Schematic representation in the Brillouin zone of electron \mathbf{k} states (filled circles) and hole states (empty circles) near the Fermi level for the CuO_2 layer in a typical cuprate. These participate in forming both Cooper pairs and CDWs. It will be noted that as expected for an even-parity d -wave superconductor, the carriers forming the Cooper pairs are of opposite spin. The electrons and holes condensing to form the CDW have the same spin as each other.

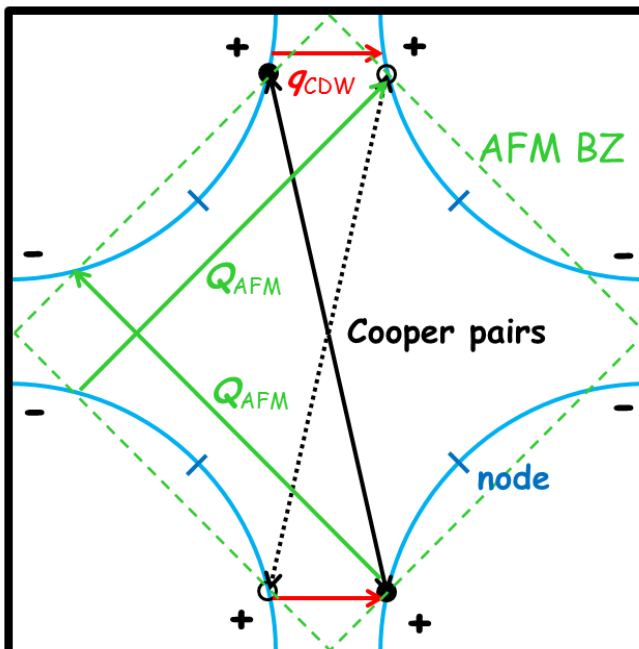


Fig. 2. As for Fig. 1, but showing two of the antiferromagnetic \mathbf{Q} -vectors which connect “hot spots”. The shape of the AFM zone boundary is also shown. The relative signs of the d -wave superconducting order parameter are marked on each of the sectors of the Fermi surface and dashes show the nodal positions where the order parameter changes sign.

gions of the FS as the doping is reduced. In this region of doping, the FS will then turn into “Fermi arcs” around the nodal regions. This is precisely the behaviour seen by photoemission (ARPES) measurements within the pseudogap region [7].

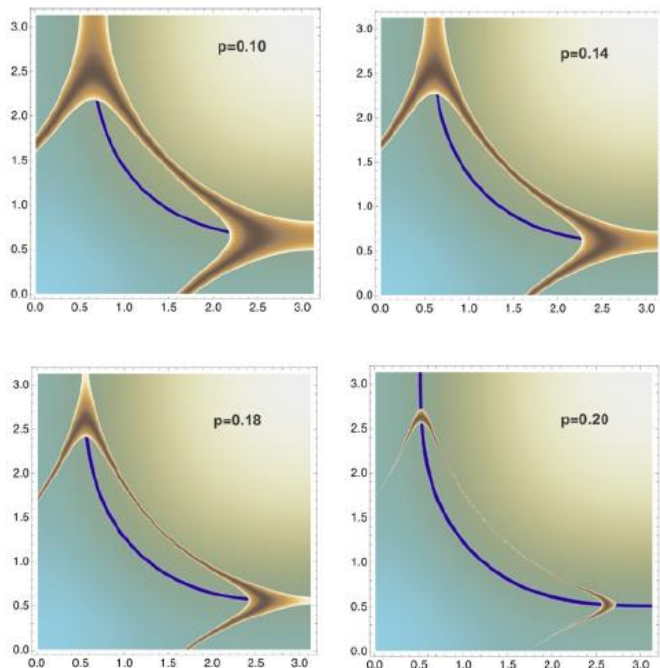


Fig. 3. Calculations of SU(2) fluctuations as a function of doping level (additional holes per unit cell varying from $p = 0.1$ to $p = 0.2$), plotted over one sector of the Fermi surface (adapted with permission from [2]). At low doping, these cover a large fraction of the states away from the nodal region.

III. FURTHER EXPERIMENTAL EVIDENCE

A. Quasiparticle relaxation time

Measurements of the quasi-particle relaxation time near T_c in underdoped materials [8] receive a very natural explanation in terms of the ideas in the SU(2) model. To explain the results, the coherent superposition of SC and CDW states is invoked, and the results cannot be explained in terms of a simple superconducting condensate. It should be noted that Ref. [8] contains a figure reminiscent of Fig. 1. However, that diagram is describing the *excitations*, which for a superconductor are a superposition of an electron and a hole. This should not be confused with the SC condensate which is composed of pairs of *like* quasiparticles.

B. CDW has a d -density wave character

The existence of CDWs has been demonstrated by many techniques, initially by nuclear magnetic resonance (NMR) [9], which could not give the actual structure, diffraction at a copper x-ray resonance [10], hard X-rays [4] and scanning tunnelling microscopy (STM) [11]. The latter experiments showed that (at least at the surface of the compounds studied) there was a “ d -density wave” – in which one pair of oxygens around a copper in a CuO_2 layer had the opposite charge modulation from the perpendicular pair. Similar features were observed in an X-ray study of the structure of the CDW in the bulk of an underdoped YBCO sample [12]. This indicates that

the local symmetries of the CDW and of the SC order parameter are very similar. All the techniques capable of spatial resolution showed that the CDW order has finite length range, particularly at low fields. The SU(2) model is expected to give effects over a range of wavevectors, which is consistent with this observation.

C. Fermi surface reconstruction

Another very prominent feature of underdoped cuprates is that the carriers exhibit a *small electron-like* FS, with a number of carriers equivalent to a small percentage of the Brillouin zone. This is demonstrated by both Hall effect [13] and quantum oscillation (QO) data [14]. It is in contrast with overdoped materials which have a *large* hole-like FS with a volume proportional to $(1+p)$, where p is the doping level, and the FS occupies more than half the Brillouin zone. There is little doubt that a Fermi surface reconstruction, associated with the CDW, causes the change in FS topology, since the \mathbf{q}_{CDW} -vectors give a quantitative account of the small FS area in a single- CuO_2 -layer material [15]. The way this occurs is illustrated in Fig.4. (The situation is more complicated, but still consistent in the

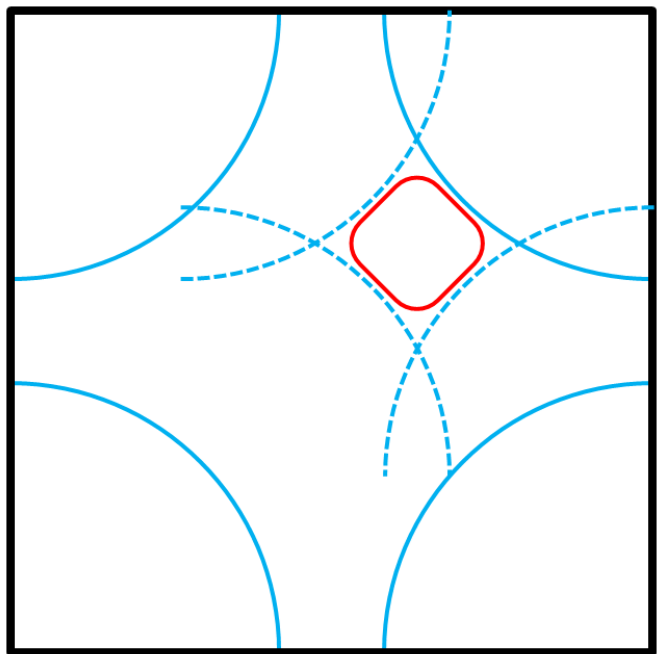


Fig. 4. Illustration of the formation of a small electron-like Fermi surface by the CDW. The carriers can pick up the \mathbf{q} of the CDW, which is equivalent to translating the hole-like surfaces (dashed in their translated positions). Where the translated surfaces overlap and are degenerate, the states can hybridise and create the small FS (red rounded square) which surrounds filled states and is therefore electron-like. We assume that the other parts of the original FS are removed by the pseudogap, i.e. that the CDW vectors connect the ends of the Fermi arcs rather than the hot spots shown in Fig. 2.

2-layer material YBCO [16].) This model for creating the small FS only operates if the CDW is biaxial, i.e. both \mathbf{a} - and \mathbf{b} -directed CDWs are present in the *same* region of space. This feature is allowed in the SU(2) picture. It should also be noted that the experimental CDW vectors do not exactly connect hot

spots. Instead, the variation of \mathbf{q}_{CDW} with doping suggests that the CDWs connect the ends of two adjacent Fermi arcs [15].

D. Pair Density Wave

Another prediction of the SU(2) model is the existence of a pair density wave (PDW) with the *same* repeat distance as the CDW. This feature has recently been detected by Josephson tunneling using an STM [17]. Since both SC & SDW share similar *d*-wave symmetry, the PDW due to their mutual interaction is expected to show close to *s* symmetry, as indeed is the case [17].

IV. COMMENTS AND QUESTIONS

All observations of CDWs show short range order in at least one of the basal plane directions. It is perhaps surprising that this can give rise to such well-defined quantum oscillation signals from the small Fermi surfaces that are believed to be associated with biaxial CDW order. The picture in Fig 4 is not reproduced by ARPES in underdoped materials; ARPES shows merely a Fermi arc, without others joining it to make a square electron-like surface. Necessarily, ARPES is a zero magnetic field measurement, whereas QO data is taken in high field, but CDWs are seen by other techniques in zero field. This remains a puzzle.

Other phenomena have been reported to occur in the pseudogap region [18, 19]. If the SU(2) picture is the fundamental cause of the pseudogap, then (for instance) Kerr rotation [18] may be a spectator effect. There are indications of intra-unit cell loop currents [19], a zero- \mathbf{q} order which has not been confirmed by NMR [9] and muon spin rotation [20] measurements, but they seem not to be universal [21].

The SU(2) picture given here does not appear to make direct contact with the rather different “stripes” seen in underdoped La-Ba & La-Sr cuprates. These stripes have elements of both CDW and AFM order. If the SU(2) theory is to succeed as a universal model for the basic physics of cuprates, this contact needs to be made. There are advocates of stripes who would claim that they represent the basic physics of all cuprates, but they also struggle to make good contact with the qualitatively different behaviour of CDW’s in the other cuprates.

It is now 31 years since the discovery of High- T_c cuprates, and although applications are finally becoming more widespread, one feels that we should soon come to a full understanding of their basic physics also. The SU(2) model is a strong contender and is continuing to develop. Refs. [2, 3] also propose how exciton-like structures may account for the “strange metal” behaviour above the SC T_c at optimal doping. Other developments of the theory may account for the Raman response associated with the superconducting state and the “resonance” seen with neutron scattering [22] and further recent applications of the theory are on the arXiv [23-25].

ACKNOWLEDGMENT

The authors would like to thank C. Pépin and M. W. Long for very useful discussions.

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