Drift of Levitated/Suspended Body in High-T_c Superconducting Levitation Systems Under Vibration—Part II: Drift Velocity for Gap Varying With Time

Xiao-Fan Gou, Xiao-Jing Zheng, and You-He Zhou

***Abstract*—To study levitation drift further, i.e., the gap between a superconductor and a permanent magnet varying with time in high-T_c superconducting levitation systems, drift velocity is introduced. Based on the numerical simulations of the dynamic response of a levitated body, and according to the essential reasons for drift, the drift velocity is first divided into two regimes: V_{ff} [related to flux flow (FF)] and V_{fc} [related to flux creep (FC)]. The drift velocity is shown to be mainly dependent upon properties of superconductivity (such as the critical current density of superconductors), initial disturbances, and applied excitations (such as the amplitude and the frequency of external excitations). Furthermore, the corresponding influences of the drift velocities V_{ff} and V_{fc} have been investigated quantitatively in this paper.**

***Index Terms*—Drift, drift velocity, gap varying, high-T_c superconducting levitation system.**

I. Introduction

As mentioned in [1], the drift of a levitated/suspended body (LB/SB) in high-T_c superconducting levitation systems is often induced by a vibration of permanent magnet (PM)/high-T_c superconductor (HTSC) or an alternating magnetic field applied to HTSC, which is directly related to the levitation stability of the systems [2], [3]. Considering of the safe operation and design of such systems, it is very important to comprehensively study the drift further.

Followed the previous study [1] on the condition of the drift occurring for the LB/SB in high-T_c superconducting levitation systems, drift velocity is introduced in this paper to study the motion velocity of the vibration center of the LB. Terentiev *et al.* [4] observed that the drift vertically down (i.e., gap decreasing with time) of an LB is induced by a vibration of PM/HTSC or an alternating magnetic field applied to HTSC, and the average drift velocity at the initial part of trajectory increases with the amplitude of applied magnetic field increasing. However, few further investigations [5], [6] on this problem have been done so far.

In this paper, the case of drift vertically down (see [1]) is selected as the example for further research. Based on [1], the aim of this paper is to numerically study the drift velocity of an LB levitated above an HTSC and the influence on it of various main factors, such as the material parameters of HTSC, the initial cooling condition for HTSC, the initial external disturbances, and the applied excitations, etc.

II. Drift Velocity

The basic equations (including the basic electromagnetic equations and the constitutive relation linking the electric field intensity $\mathbf{E}$ and the current density $\mathbf{J}$) and the numerical procedure in this paper are the same as those in [1]. In addition, the examination of the numerical code is also adopted as the same as that in [1]. So, these necessary equations and processes are excluded in this paper. The geometric and material parameters used in the following simulations are listed in Table I, some of which are the same as those given in Yoshida *et al.* [7]. In addition, other parameters are the same as those in [1] in this series.

As observed by Terentiev *et al.* [4], in the process of drift of LB, the gap between the LB and the HTSC decreases with time, or the spatial location of the LB from HTSC descends with time. So, to quantitatively describe this variation, it is very necessary to define a drift velocity of the LB. Obviously, the drift velocity is directly related to the displacement of the vibration center of the LB varying with time [4]. Before its specific definition is introduced, the dynamic response under free and forced vibration, in which the drift can be characterized for the different parameters of the systems, have been simulated numerically, as shown in Figs. 1–3. The results show various main influences on time-variation of the displacement of the vibration center of the LB. Fig. 1 displays the dynamic response of the LB under free vibration for different initial velocities. With time spanning, it is shown that the vibration center of the LB drops down no matter what initial velocity is achieved; the larger the initial velocity, the faster is the variation of the displacement of the vibration center with time in the initial period (about $\leq$0.2 s), and after the initial period (about $>$0.2 s), the variation of the displacement at the different initial velocities are almost same. Fig. 2

Manuscript received February 9, 2006; revised November 10, 2006. This work was supported by the Fund of Pre-Research for Key Basic Researches of the Ministry of Science and Technology of China, by the Fund of the National Science Foundation of China (10472038), by the doctoral fund of the Ministry of Education of China (20050730016), by the Youth Fund of the National Science Foundation of China (10502019), China Postdoctoral Science Foundation, and by the Postdoctoral Research Project of Jiangsu Province. This paper was recommended by Associate Editor P. J. Masson.

X.-F. Gou is with the Department of Engineering Mechanics, Hohai University, Nanjing, Jiangsu 210098, China (e-mail: xfgou@163.com).

X.-J. Zheng and Y.-H. Zhou are with the Department of Mechanics, Lanzhou University, Lanzhou, Gansu 730000, China (e-mail: xjzheng@lzu.edu.cn; zhouyh@lzu.edu.cn).

Digital Object Identifier 10.1109/TASC.2007.902105

TABLE I
GEOMETRICAL AND PHYSICAL PARAMETERS EMPLOYED IN SIMULATION

	Diameter (mm)	Thick (mm)	J_c (A/m^2)	E_c (V/m)	U_0 (eV)	ρ_f (Ωm)	Residual field (Tesla)
HTSC	18	2.5	5.0×10^{7}	1.0×10^{-4}	0.1	5.0×10^{-10}	
PM	25	22.5					0.4

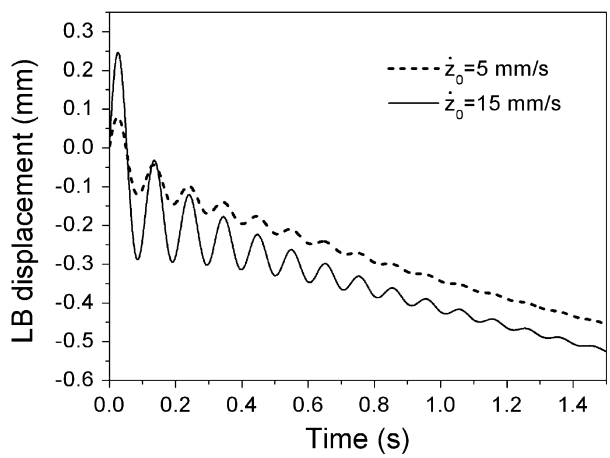


Fig. 1. Dynamic response of LB under free vibration for different initial velocities. (Initial cooling height 15 mm; static equilibrium gap = 3 mm).

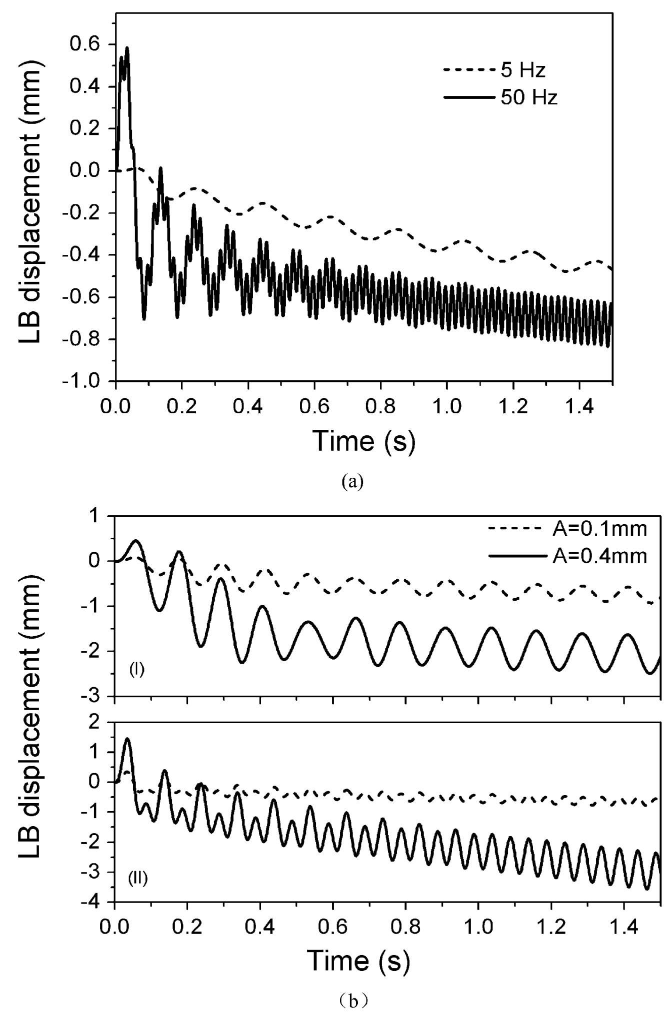


Fig. 2. Dynamic response of LB under forced vibration for different (a) excitation frequencies ($A = 0.1$ mm) and (b) excitation amplitudes. (Initial cooling height 15 mm; static equilibrium gap = 3 mm; initial velocity of LB $\dot{z}_0 = 0$).

also presents the dynamic response of the LB under forced vibration for the different excitation frequencies ($A = 0.1$ mm) and the excitation amplitudes ($f_a = 8$ Hz or $f_a = 20$ Hz). Results similar to the ones in Fig. 1 are found: in the initial period (about $\leq$0.2 s), the larger the applied excitation frequency, the faster the variation of the displacement of the vibration center with time; but with time spanning, the variations for different applied excitation frequencies are almost the same [see Fig. 2(a)]. The results indicate that the influence of the excitation frequency on drift exists only in the initial period of the dynamic response of the LB. In two cases of different excitation amplitudes [see Fig. 2(b)], if the applied excitation frequency is lower than the natural frequency (here the natural frequency at static equilibrium gap 3 mm is about 10 Hz estimated from the magnetic stiffness at this location) of this levitation system, in the initial period (also about $\leq$0.2 s), the larger the applied excitation amplitude, the faster the variation of the displacement of the vibration center with time; but with time spanning, the variations for different applied excitation frequencies are almost same [see Fig. 2(b) (I)]. While the applied excitation frequency is higher than the natural frequency of this system, the displacement always increases with the applied excitation amplitude enlarging [see Fig. 2(b) (II)]. Thus, these results show that the influence of the excitation amplitude on drift exists only in the initial period on condition that the excitation frequency is less than the natural frequency of this system, and in the whole response on condition that the excitation frequency is more than the natural frequency of this system.

From the $\mathbf{E}-\mathbf{J}$ constitutive relation of flux flow and creep model (see [1, eq. (7)] or Fig. 4), the material parameters of HTSC E_c and U_0 are closely related to flux creep, and ρ_f is closely related to flux flow in HTSC. To explore the essential reason of drift in high-T_c superconducting levitation systems, the dynamic response of LB under free vibration in two cases of different material parameters of HTSC has been simulated and shown in Fig. 3. Obviously, the time-variations of the displacement of the vibration center are different for the different material parameters of HTSC. In these results, the more important ones are that the influence of the material parameters E_c and U_0 displays mostly after the initial period [also about >0.2 s, see Fig. 3(a) and (b)], while that of the parameter ρ_f does in the initial period [the period is closely related to the ρ_f, see Fig. 3(c)].

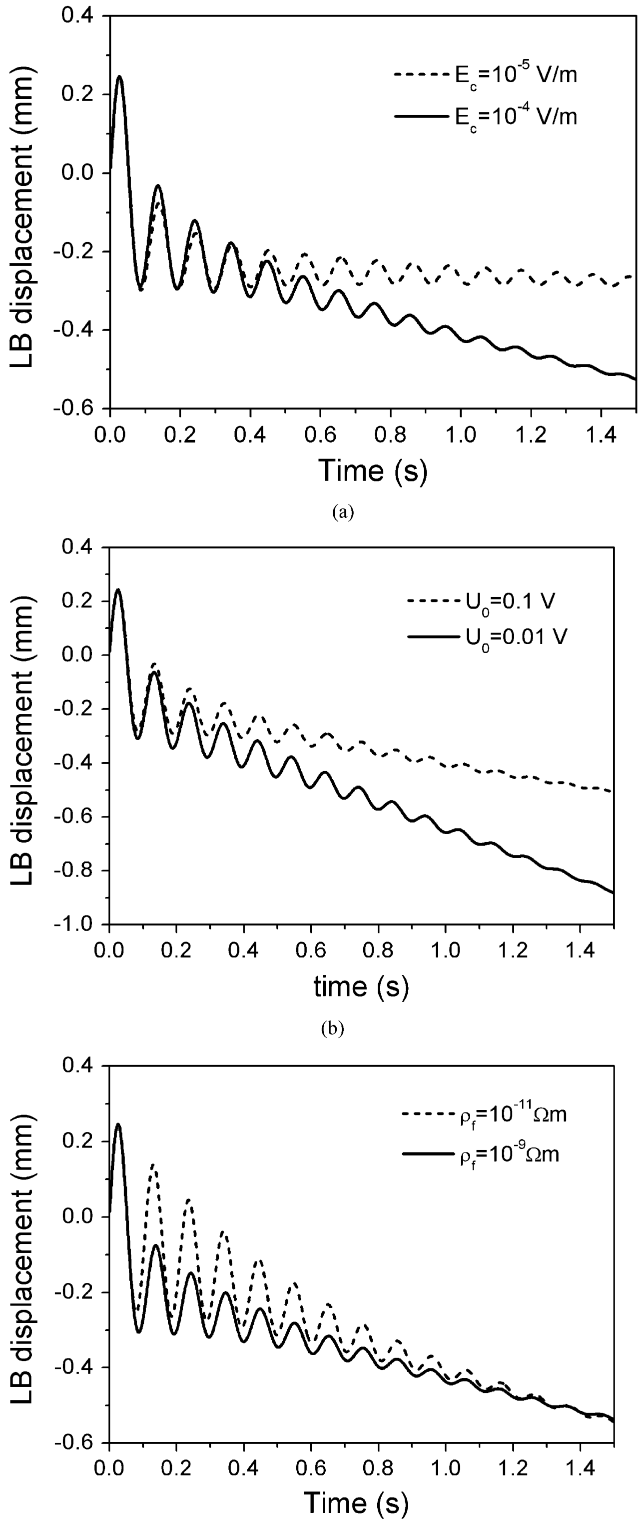


Fig. 3. Dynamic response of LB under free vibration for different material parameters of HTSC. (a) E_c; (b) U_0; (c) ρ_f. (Initial cooling height 15 mm; initial velocity of LB $\dot{z}_0 = 15$ mm/s.)

Besides, the data also show that the larger/smaller the value of E_c/U_0 is, the faster the variation of the displacement of the vibration center is [see Fig. 3(a) and (b)], and the larger the value of ρ_f is, the faster the variation of the displacement of the vibration center is [see Fig. 3(c)]. Thus, it can be concluded that LB's drifting in such levitation systems in essence results from flux motion (flux flow and creep) in HTSC.

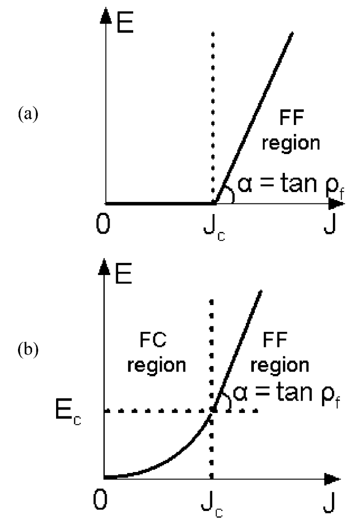


Fig. 4. Diagram of macro-models of HTSCs. (a) Flux flow model; (b) flux flow and creep model.

From the above discussion, it can be found that the two distinct processes exist in the dynamic response of the LB: in the initial period (about $\leq$0.2 s), the time-variation of the displacement of the vibration center of the LB is rapid, and after the initial period, the time-variation is stable and slower than that in the initial period. As is well known, flux creep is thermally activated, while flux flow is often induced by some disturbances; and in general, flux flow and creep exist in superconductors at the same time, but at the different period only one dominates flux motion [8], [9]. Connecting with Fig. 3, we can conclude that flux flow (denoted by ρ_f) is a dominator in the initial period of the dynamic response of the LB, while flux creep (denoted by E_c and U_0) is a dominator instead after the initial period on condition that the applied excitation is smaller (e.g., $A = 0.1$ mm). Thus, we divide the drift velocity into two regimes: in the initial period, the drift velocity is denoted by V_{ff} [related to flux flow (FF)]; while after the initial period, the drift velocity is denoted by V_{fc} [related to flux creep (FC)]. For simplicity, V_{ff} is regarded as the average velocity in the initial period of $0 \sim \tau_m$, and V_{fc} is regarded as the average velocity in the period of $\tau_m \sim \tau$. So, these specific definitions are given as

$$V_{\mathrm{ff}} = \frac{\left| z|_{\mathrm{time}=\tau_m} - z|_{\mathrm{time}=0} \right|}{\tau_m} \quad (1)$$

and

$$V_{\mathrm{fc}} = \frac{\left| z|_{\mathrm{time}=\tau} - z|_{\mathrm{time}=\tau_m} \right|}{\tau - \tau_m} \quad (2)$$

where $z|_{\mathrm{time}}$ is the displacement of vibration center of the LB relative to its static equilibrium position (see [1, Fig. 4], $z = Z - Z_{\mathrm{eq}} = 0$) at the time; τ is the given time when the vibration of the LB becomes more stable, and assigned as the unified value 2 s for simplicity under the parameters in this paper; τ_m

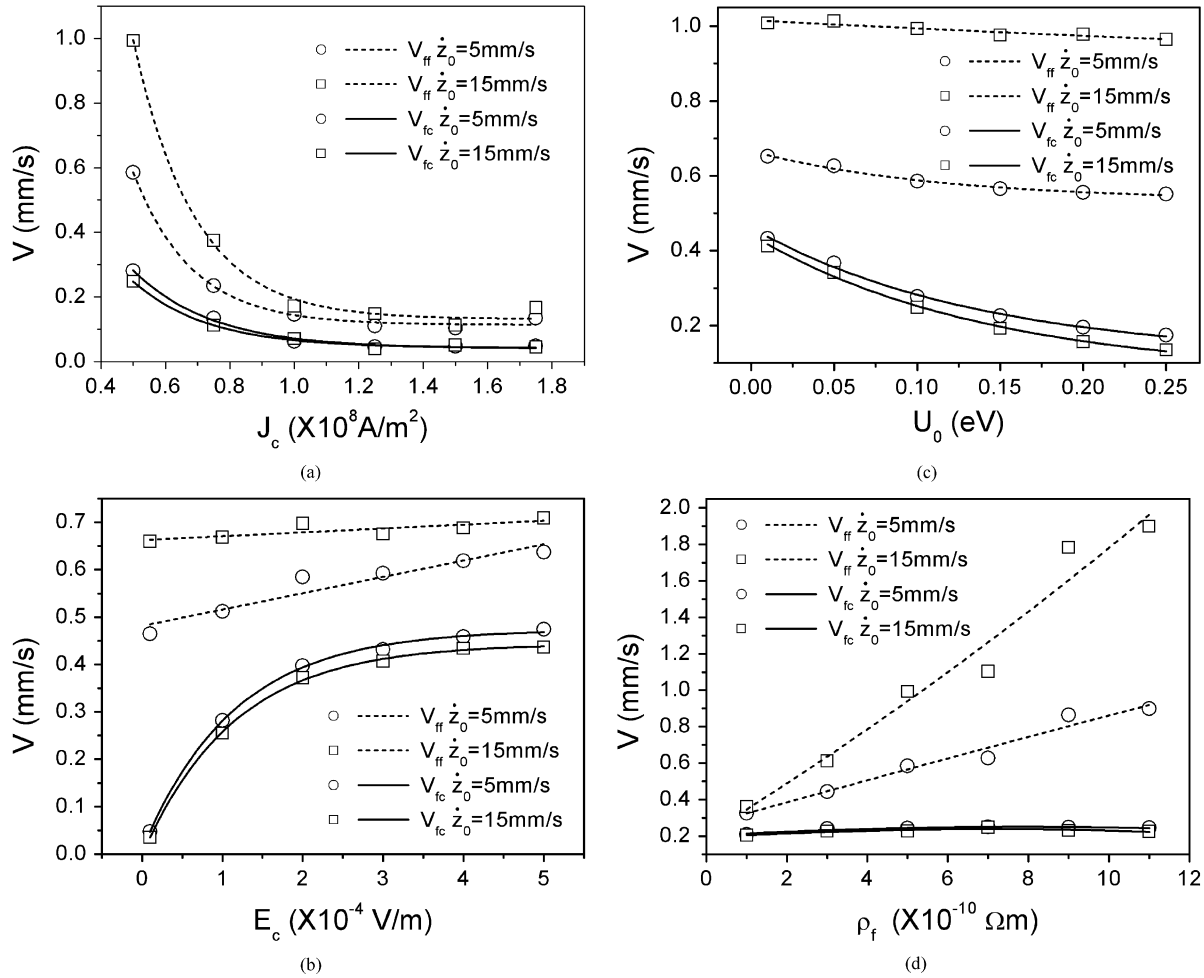


Fig. 5. Material parameters of HTSC dependence of the drift velocity of LB under free vibration for different initial velocities. (a) J_c; (b) E_c; (c) U_0; (d) ρ_f (initial cooling height 15 mm).

is the magnetic diffusion time in single domain YBCO, and approximately given by [10], [11]

$$\tau_m = \frac{\mu_0 a^2}{\rho_f} \tag{3}$$

where μ_0 is the permeability of free space; a is the characteristic dimension of the superconductors, typically some thickness or length over which diffusion occurs. In this levitation system, a is considered the radius of the cylindrical HTSC. ρ_f is the flow resistivity as that in the flux flow and creep model. Obviously, the τ_m is varying with ρ_f for different HTSC samples.

III. Results and Discussion

Based on the definition of the drift velocity above, the influences of various main factors on the drift velocity V_{ff} and V_{fc} have been studied quantitatively in this paper. Material parameters of HTSC dependence of the drift velocity under free vibration in two cases of different initial velocities has first been shown in Fig. 5. In general, for different material parameters of HTSC, the magnetic force exerted on the PM (or HTSC) at the same gap are not different. So, for an LB (including a PM/HTSC and an additional load) with a given weight, there exist different static equilibrium positions. In this paper, the weight of LB is given as the same as the magnetic force for gap 3 mm and initial cooling height 15 mm under the parameters given in Table I. Fig. 5(a) displays that the critical current density J_c of HTSC dependence of the drift velocity in two cases of different initial velocities. Obviously, the drift velocities V_{ff} and V_{fc} decrease with the critical current density J_c, and approach zero little by little when the critical current density J_c is more than $1.5 \times 10^8\mathrm{A/m}^2$. In general, the bigger critical current density J_c means that flux pinning in HTSC is stronger, or the influence of flux motion is weaker. As a result, the drift of the LB is unapparent. In addition, the value of the V_{ff} is always larger than that of the V_{fc}, and also, the influence of the initial velocity on the V_{ff} is more apparent and stronger: the bigger the initial velocity, the bigger the V_{ff}, while the influence on the V_{fc} is weaker [also shown in Fig. 5(b) and (c)]. As the impotent parameters E_c and U_0 in the constitutive relation of HTSC which describe flux creep in HTSC, their influence on the drift velocity in two different initial velocities has also been simulated as shown in Fig. 5(b) and (c). Besides some similar characteristics to those shown in Fig. 5(a), it is shown obviously that the curve tendency of the variation of the drift velocity with the E_c is opposite to that with the U_0. The reason consists in different characterizations of flux creep in the constitutive relation: bigger E_c and smaller U_0 means

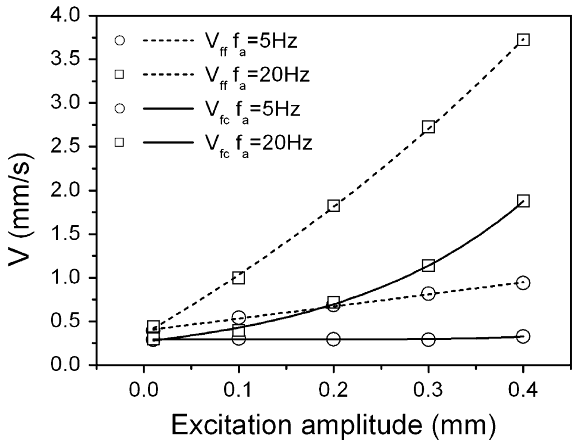


Fig. 6. Excitation amplitude dependence of the drift velocity of LB under forced vibration for different excitation frequencies. (Initial cooling height 15 mm; static equilibrium gap = 3 mm).

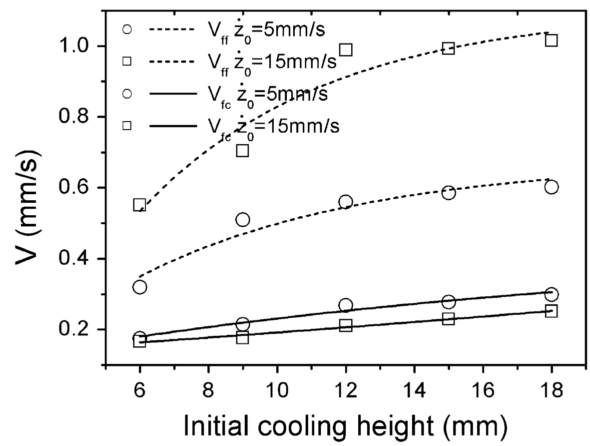


Fig. 7. Initial cooling height dependence of the drift velocity of LB under free vibration for different initial velocities.

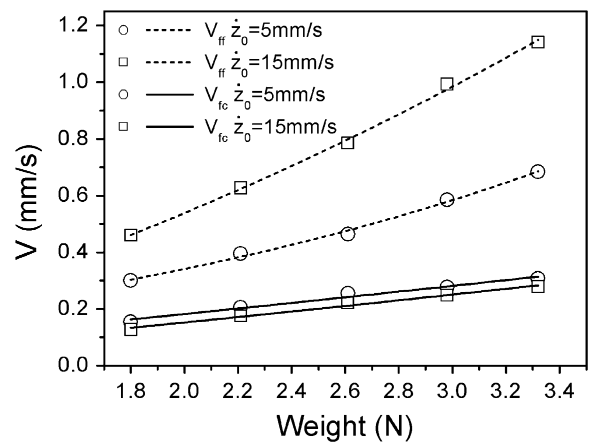


Fig. 8. Weight of LB dependence of the drift velocity of LB under free vibration for different initial velocities. (The initial cooling height 15 mm).

intenser flux creep in HTSC. So, as a result, the drift velocity $V_{\rm fc}$ increases. In addition, the curve tendency of the variation of the drift velocity $V_{\rm ff}$ is also similar to the variation of the drift velocity $V_{\rm fc}$. From the $\mathbf{E}-\mathbf{J}$ constitutive relation of flux flow and creep model, though flux creep in HTSC is characterized by the E_c and U_0, whose varying also induces additional flux flow. As mentioned above, flux flow in HTSC is characterized by the material parameter ρ_f; its influence on the drift velocity in two different initial velocities is shown in Fig. 5(d). It should be noted that the τ_m is varying with ρ_f according to (3), which has been considered in simulation. Besides some similar results as shown in Fig. 5(a)–(c), there is an obvious increasing of the drift velocity $V_{\rm ff}$, while almost no variation of the drift velocity $V_{\rm fc}$ with ρ_f.

Under forced vibration, applied excitation amplitude dependence of the drift velocity in two cases of different excitation frequencies has been further simulated as shown in Fig. 6. Connecting with Fig. 2, one can find that, when the excitation frequency is higher than the natural frequency of this system (for example 20 Hz), the drift velocities $V_{\rm ff}$ and $V_{\rm fc}$ increase with the excitation amplitude; while when the excitation frequency is lower than the natural frequency of this system (for example, 5 Hz), with the excitation amplitude increasing, only the $V_{\rm ff}$ increases slightly, but the $V_{\rm fc}$ does not almost vary.

For the different initial cooling conditions (i.e., initial cooling heights), the magnetic forces at the same levitation gap are not the same. As a result, for the LB with a given levitated weight (here, for example, also adopted as the magnetic force for gap 3 mm/s and initial cooling height 15 mm under the parameters given in Table I), its levitation gap (or static equilibrium position) is also different from each other. So, one can guess that there is a distinct difference in the dynamic responses of the LB at different levitation gaps. Under free vibration, initial cooling height dependence of the drift velocity in two cases of different initial velocities is shown in Fig. 7. Besides the similar results those above, there is an obvious increase of all the drift velocities $V_{\rm ff}$ and $V_{\rm fc}$ with the initial cooling height.

For a given initial cooling condition, it is apparent that, at different levitation gaps, the magnetic forces are different, and in other words, the weights levitated by HTSC are different. Thus, the dynamic responses of the LB with different weights are different. The weight of the LB dependence of the drift velocity under free vibration in two cases of different initial velocities is shown in Fig. 8. It is also obvious that, besides the similar results to the above, the drift velocities $V_{\rm ff}$ and $V_{\rm fc}$ increase rapidly with the weight of the LB.

IV. Conclusion

To study levitation drift of the LB levitated above HTSC further, drift velocity has been introduced. Based on the numerical simulations of the dynamic response of the LB levitated above HTSC, and in consideration of the essential reasons for drift, we first divided drift velocity into two regimes: $V_{\rm ff}$ and $V_{\rm fc}$. On these bases, the conclusions are summarized as follows:

1) Under free vibration, the influence of the initial velocity of the LB on the $V_{\rm ff}$ is very remarkable, but on the $V_{\rm fc}$ is almost unapparent. The $V_{\rm ff}$ is increasing with the initial velocity.
2) Under forced vibration, the influence on drift velocity is closely related to the applied frequency: when the frequency is higher than the natural frequency of this system, all the $V_{\rm ff}$ and $V_{\rm fc}$ increase remarkably with the applied

amplitude, and when the frequency is lower than the natural frequency, with the applied amplitude increasing the $V_{\rm ff}$ increases, but the variation of the $V_{\rm fc}$ is almost unapparent.

3) The influence of material parameters of HTSC on drift velocity is more essential. With the critical current density J_c increasing, all the drift velocities $V_{\rm ff}$ and $V_{\rm fc}$ decrease remarkably; with the E_c/U_0 increasing/decreasing, all the drift velocities $V_{\rm ff}$ and $V_{\rm fc}$ increase remarkably, and with the ρ_f increasing, the $V_{\rm ff}$ increases remarkably, but the $V_{\rm fc}$ is almost unvaried.

ACKNOWLEDGMENT

The authors would like to thank the reviewers for their valuable suggestions in the revisions.

REFERENCES

[1] X.-F. Gou, X.-J. Zheng, and Y.-H. Zhou, "Drift of levitated/suspended body in high-T_c superconducting levitation systems under vibration—Part I: A criterion based on magnetic force-gap relation for gap varying with time," *IEEE Trans. Appl. Supercond.*, 2007.

[2] F. C. Moon, *Superconducting Levitation*. New York: Wiley, 1994.

[3] J. R. Hull, "Topic Review: Superconducting bearing," *Supercond. Sci. Technol.*, vol. 13, pp. R1–R15, 2000.

[4] A. N. Terentiev and A. A. Kuznetsov, "Drift of levitated YBCO superconductor induced by both a variable magnetic field and a vibration," *Physica C*, vol. 195, pp. 41–46, 1992.

[5] T. Hikihara and F. C. Moon, "Levitation drift of a magnet supported by a high-T_c superconductor under vibration," *Physica C*, vol. 250, pp. 121–127, 1995.

[6] T. A. Coombs and A. M. Campbell, "Gap decay in superconducting magnetic bearings under the influence of vibrations," *Physica C*, vol. 256, pp. 298–302, 1996.

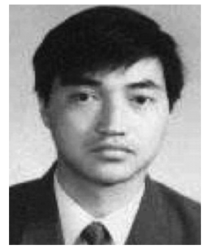
[7] Y. Yoshida, M. Uesaka, and K. Miya, "Evaluation of dynamic magnetic force of high-T_c superconductor with flux flow and creep," *Int. J. Appl. Electromagn. Mater.*, vol. 5, pp. 83–89, 1994.

[8] H. Ullmaier, *Irreversible Properties of Type-II Superconductors.*. New York: Springer-Verlag, 1975.

[9] P. W. Anderson, "Theory of flux creep in hard superconductors," *Phys. Rev. Lett.*, vol. 9, pp. 309–311, 1962.

[10] Y. S. Cha, "Magnetic diffusion in high-T_c superconductors," *Physica C*, vol. 330, pp. 1–8, 2000.

[11] T. R. Askew, J. C. O'Daniel, J. M. Weber, and Y. S. Cha, "Nonlinear magnetic diffusion in single domain YBCO cylinders under pulsed field conditions," *IEEE Trans. Appl. Supercond.*, vol. 15, no. 2, pt. 3, pp. 3758–3761, Jun. 2005.

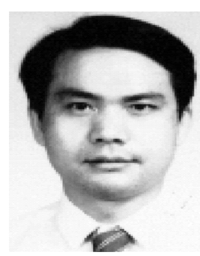


Xiao-Fan Gou received the Ph.D. degree in superconductivity from Lanzhou University, Lanzhou, Gansu, China, in 2004.

In 2006, he was appointed an Associate Professor in the Department of Engineering Mechanics, Hohai University, Nanjing, Jiangsu, China. Now his research interests are in the numerical analysis of electromagnetic phenomena in superconductors, including the application of superconductors to levitation.

Xiao-Jing Zheng received the Masters Degree in Engineering Science from Huazhong University of Science and Technology, Wuhan, Hubei, China, in 1984, and the Ph.D. degree from Lanzhou University, Lanzhou, Gansu, China, in 1987.

Since 1991, she has been a Professor in the Department of Mechanics, Lanzhou University, Gansu, China. Her research interests include analysis of mechanics of multifield coupling, electro-magneto-solid mechanics, and electrification mechanism of aeolian sand grains, etc. She has published about 140 papers and two text books in her research areas.



You-He Zhou received the Masters Degree in Engineering Science from Huazhong University of Science and Technology, Wuhan, Hubei, China, in 1984, and the Ph.D. degree from Lanzhou University, Lanzhou, Gansu, China, in 1989.

He was appointed a Professor of the Department of Mechanics, Lanzhou University, Lanzhou, China, in 1996. His research interests include dynamic analysis, electro-magneto-solid mechanics, and dynamic control of smart structures, etc. He has published about 160 papers and one textbook in his research areas.