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IEEE-CSC, ESAS and CSSJ SUPERCONDUCTIVITY NEWS FORUM (global edition), Issue No. 56, March, 2024. Invited presentation given at ISS 2023, Nov. 2023, Wellington, New Zealand

Effects of Strong Capacitive and Inductive Coupling on Hysteretic rf SQUID Metamaterials

Jingnan Cai, Steven M. Anlage

Collaborators: Thomas Antonsen, Edward Ott (Univ. of Maryland) Alexander Zhuravel (Kharkov, Ukraine) Alexey Ustinov (Karlsruhe, Germany) Johanne Hizanidis, Nikos Lazarides and George Tsironis (Univ. of Crete, Greece)

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LSM image of 2D rf SQUID metamaterial



3D rf SQUID metamaterial





- Brief Review of Superconducting Metamaterials
- rf SQUID Metamaterials
 - Tuning
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 - Laser Scanning Microscopy
- 3D Stacked rf SQUID Metamaterials
 - The Role of Capacitive Coupling
- Current / Future Work
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Motivation and Background

Artificial Structures made up of "meta-atoms" with <u>new</u> or <u>extreme</u> properties

Collections of sub-wavelength scatterers that create an effective medium with new functionality

Enhance light/matter interactions



Functional Metasurfaces



https://www.photonics.com/Articles/Metasurfaces Open Up Research Paths For Ouantum/a68324



Why <u>Superconducting</u> Metamaterials?

Many exciting applications of metamaterials: Metasurface optics Cloaking Super-resolution imaging, etc. ...



... have strict <u>REQUIREMENTS</u> on the <u>metamaterials</u>:

Ultra-Low Losses

Ability to scale down in size (e.g. $\lambda/10^2$) and texture the "meta-atoms"

Nonlinearity with wide and fast tunability of the index of refraction *n*

- ... and superconductors bring these <u>new features</u> to the metamaterials field:
 - Strong diamagnetism
 - Flux quantization and Josephson effects
 - Quantized energy states and quantum interactions with light

M. Ricci, N. Orloff, S.M.A., "Superconducting Metamaterials," Appl. Phys. Lett. 87, 034102 (2005)
S.M.A. "The Physics and Applications of Superconducting Metamaterials," J. Opt. 13, 024001 (2011)
P. Jung, A. V. Ustinov, and S.M.A., "Progress in Superconducting Metamaterials," Supercond. Sci. Technol. 27, 073001 (2014)
N. Lazarides and G. P. Tsironis, "Superconducting Metamaterials," Physics Reports 752, 1 (2018)



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How to make them: Step 1





Thanks to P. Kneisel @ Jefferson Lab



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How to make them: Step 2











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split-ring resonator

(used to create

'optical magnetism')

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SQUID = Superconducting QUantum Interference Device A self-resonant meta-atom with very nonlinear properties Resonant Frequency of rf SQUID







rf SQUID Meta-Atoms

Josephson Inductance is large, tunable and nonlinear





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rf SQUID Superconducting Metamaterial $\lambda \gg r, d$ • Low loss • Small Size a $-\lambda \sim 3$ cm (~ 10 GHz) $-2r = 20 \sim 800 \ \mu m$ rf SQUID meta-atoms

The SQUIDs interact by means of dipole – dipole coupling

→ Collective Behavior

Original theory proposals: C. Du, H. Chen, and S. Li, PRB <u>74</u>, 113105 (2006) N. Lazarides and G. P. Tsironis, APL <u>90</u>, 163501 (2007)



Measurement of rf SQUID Metamaterial











Applications of <u>Nonlinear</u> SQUID Metamaterials

Quantum-Limited Amplifiers





A near-quantum-limited Josephson traveling-wave parametric amplifier

C. Macklin,^{1,2}* K. O'Brien,³ D. Hover,⁴ M. E. Schwartz,¹ V. Bolkhovsky,⁴ X. Zhang,^{3,†} W. D. Oliver,^{4,7} I. Siddiqi¹



A wideband, low-noise superconducting amplifier with high dynamic range

Byeong Ho Eom¹, Peter K. Day²*, Henry G. LeDuc² and Jonas Zmuidzinas^{1,2}





Two-tone spectroscopy of a SQUID metamaterial in the nonlinear regime

E. I. Kiselev,^{1,2} A. S. Averkin,³ M. V. Fistul,^{4,3,5} V. P. Koshelets,⁶ and A. V. Ustinov^{2,3,5}

Phys. Rev. Research <u>1</u>, 033096 (2019)





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Long-Range Inductive Coupling in rf SQUID metamaterials

Side-by-side SQUIDs have mutual inductance *M* that falls off like a dipole-dipole interaction: M < 0, with $|M| \sim \frac{1}{r^3}$



Sample edges are also a large perturbation

Chimera: coexistence of synchronous and asynchronous groups of oscillations in a material, even for uniform constituent atoms and symmetric couplings

PHYSICAL REVIEW E 94, 032219 (2016)

Robust chimera states in SQUID metamaterials with local interactions

J. Hizanidis, N. Lazarides, and G. P. Tsironis



Other theory papers motivated by our rf SQUID metamaterials: A. Banerjee and D. Sikder, Phys Rev E <u>98</u>, 032220 (2018). N. Lazarides and G. P. Tsironis, Physics Reports <u>752</u>, 1 (2018). J. Hizanidis, Front. Appl. Math. Stat. <u>5</u>, 33 (2019). N. Lazarides, Chaos, Solitons and Fractals <u>130</u>, 109413 (2020) J. Shena, Chaos <u>30</u>, 123127 (2020) J. Shena, Chaos <u>31</u>, 093102 (2021) PHYSICAL REVIEW B 91, 054303 (2015)

Chimeras in SQUID metamaterials

N. Lazarides,^{1,2} G. Neofotistos,¹ and G. P. Tsironis^{1,2,3}







Imaging experiments done by:A. P. Zhuravel in laboratory of A. Ustinov, KIT, GermanyAlso Seokjin Bae and Jingnan Cai at UMD



Origins of rf Photoresponse



Heating of JJ leads to decrease of critical current and shift of resonance

$$L_{JJ} = \frac{\Phi_0}{2\pi I_c(T) \cos \delta}$$
 Photoresponse ~ $|\mathbf{I}_{rf}|^2$ in junction



IEEE-CSC, ESAS and CSSJ SUPERCONDUCTIVITY NEWS FORUM (global edition), Issue No. 56, March, 2024. Invited presentation given at ISS 2023, Nov. 2023, Wellington, New Zealand Imaging rf SQUID Metamaterial High-Power Coherent Mode Reflectivity LSM image 27 x 27 rf SQUID metamaterial Photoresponse $\sim |I_{rf}|^2$ PhotoResponse Imaging of rf SQUID Metamaterial Source Simulation Results Simulated $|I_{rf}|$ in each SQUID High Power (-20 dBm) attenuator Coherent Mode Input rf wave Waveguide 17.9 GHz Cryogenic environment **RF SQUID** metamaterial T=4.4 K Scanning Erft Mirrors $B_{rf} \leftarrow$ output rf wave Lock-in Amplifier Laser Detector A. P. Zhuravel, P. Jung, S. Anlage, A. Ustinov, KIT, Germany RF PhotoResponse LSM image



Imaging rf SQUID Metamaterial The "Dark Modes" are now visible!

A weak global driving field reveals strong disorder of the sample



Images taken at low rf flux amplitude $\Phi_{rf} = 10^{-4} \Phi_0$

 $\Phi_{rf} = 10^{-3} \, \Phi_0$

Stronger global rf flux creates a coherent mode





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 $\mathcal{M} > 0$

Next-Generation rf SQUID Metamaterials Move into the third dimension



side-coupled rf SQUIDs Study 3D "stacked metamaterials" A. M. Zagoskin J. Optics <u>14</u>, 114011 (2012) J. Shena, *et al.*, Chaos <u>30</u>, 123127 (2020)

Positive (Ferromagnetic) Coupling between meta-atoms $(M/L \rightarrow -0.2 \text{ to } 0 \text{ to } +0.70 \text{ as overlap increases})$





Next-Generation rf SQUID Metamaterials Move into the third dimension







side-coupled rf SQUIDs

2D metamaterial with mixed ferromagnetic and anti-ferromagnetic coupling

Thanks to Robin Cantor @StarCryoelectronics







Josephson Junction Fabrication is Highly Constrained!



The blue layer wiring is on top of the orange layer wiring

Note that large capacitors are formed where the blue layer wiring crosses the orange layer wiring

 C_{ov}

Note that *dc currents* only flow through the **blue** or **orange** loops

rf currents can also flow <u>between</u> the <u>blue</u> and <u>orange</u> loops



 $C_{ov} \approx C$, so the metamaterial dynamics are strongly altered!



Overlapping SQUID geometry Focus on a Corner-Coupled SQUID Pair





The JJs can only be fabricated on the base layer



Single SQUID Equations for $\delta(t)$ Flux Quantization vs. Faraday's Law Approach

rf SQUID equation utilizing flux quantization and RCSJ model for current

 $\Phi = \Phi_{app} + \Phi_{ind}$

Total flux quantization: $2\pi n = \delta + \frac{2e}{\hbar} \Phi$,

Combine and use the RCSJ model for $\Phi_{ind} = LI$:

$$\Phi_{dc} + \Phi_{rf} \sin \omega t = \frac{\Phi_0}{2\pi} \,\delta + L \left(I_c \,\sin \delta + \frac{\Phi_0}{2\pi} \frac{\dot{\delta}}{R} + C \frac{\Phi_0}{2\pi} \ddot{\delta} \right)$$

Second-order nonlinear differential equation for $\delta(t)$

Voltage on the junction: $V = \frac{\Phi_0}{2\pi} \dot{\delta}$ rf SQUID equation utilizing Faraday's law

$$V = \frac{d}{dt} \left(\Phi_{app} + \Phi_{ind} \right)$$

Faraday's law applied to a single SQUID results in the time-derivative of the rf SQUID equation

$$\frac{d}{dt} \left[\Phi_{dc} + \Phi_{rf} \sin \omega t \right] = \frac{d}{dt} \left[\frac{\Phi_0}{2\pi} \,\delta + L \left(I_c \,\sin \delta + \frac{\Phi_0}{2\pi} \frac{\dot{\delta}}{R} + C \frac{\Phi_0}{2\pi} \ddot{\delta} \right) \right]$$





Two Corner-Coupled SQUID Equations

Faraday's Law / Voltage Approach

Now there are voltage drops in the SQUID loops that are <u>not</u> simply related to $\dot{\delta}$

2-SQUID flux relation:

$$egin{pmatrix} \Phi^{app}_{a} \ \Phi^{app}_{b} \end{pmatrix} = rac{\Phi_0}{2\pi} egin{pmatrix} \delta_a \ \delta_b \end{pmatrix} + egin{pmatrix} \Phi^{ind}_a \ \Phi^{ind}_b \end{pmatrix}$$

Apply Faraday's law to the central loop:

$$V_b - V_1 + V_2 = \frac{d}{dt} \left(\Phi_{cen}^{app} - \Phi_{cen}^{ind} \right)$$

where V_1 and V_2 are the voltages across the capacitors, V_b is the voltage across junction b

Conservation of current through the overlap capacitors:



The flux relations become:

$$\begin{pmatrix} \Phi_{dc} + \Phi_{rf} \sin(\omega t) \\ \Phi_{dc} + \Phi_{rf} \sin(\omega t) \end{pmatrix} = \frac{\Phi_0}{2\pi} \begin{pmatrix} \delta_a \\ \delta_b \end{pmatrix} + \\ \begin{pmatrix} L_{geo} & M \\ M & L_{geo} \end{pmatrix} \begin{pmatrix} I_{a0} \\ I_{b0} \end{pmatrix} + C_{ov} \begin{pmatrix} -L_{\delta a} \dot{V}_1 \\ L_{\delta b} \dot{V}_1 \end{pmatrix}$$

with

 $\dot{V}_1 = [(1 + \kappa_{vb})\dot{V}_b + \kappa_{va}\dot{V}_a + L_{Ib}\ddot{I}_b + L_{Ia}\ddot{I}_a$ $- \ddot{\Phi}^{app}_{cen} - \kappa_{va}\ddot{\Phi}^{app}_a - \kappa_{vb}\ddot{\Phi}^{app}_b]/2$

and $L_{\delta a}$, $L_{\delta b}$ are partial inductances

This is a pair of coupled 4th-order nonlinear equations for δ_a , δ_b . These can be reduced to a set of 6 first-order equations (utilizing constraints)

Write $\delta = \delta_{dc} + \delta_{rf} e^{i\omega t}$, and linearize by assuming $|\delta_{rf}| \ll 1$, results in a characteristic equation for ω of 6th order, yielding 3 positive and 3 negative frequency solutions.



IEEE-CSC, ESAS and CSSJ SUPERCONDUCTIVITY NEWS FORUM (global edition), Issue No. 56, March, 2024. Invited presentation given at ISS 2023, Nov. 2023, Wellington, New Zealand Resonant Modes of Four Corner-Coupled rf SQUIDs



https://doi.org/10.48550/arXiv.2402.07044

NERSIT,

Resonant Modes of $N \times N \times 2$ **Corner-Coupled rf SQUIDs**

 $2N^2$ SQUID loops $(2N - 1)^2$ partial loops $2(N - 1)^2$ extra SQUID loops

Total: $8N^2 - 8N + 3$

Gauge-invariant phase differences $\delta: 2N^2$

Each loop contributes one equation, giving a total number of equations: $8N^2 - 8N + 3$

There are a total of $8N^2 - 8N + 3$ unknowns, resulting in a **total number of modes**: $8N^2 - 8N + 3$



N = 2 case 19 modes



Next-Generation rf SQUID Metamaterials Compare 2D and 3D samples made up of the same rf SQUIDs



Left: SNAP161D 12×12×1 rf SQUID array

 $\beta_{\rm rf} = 5.48 @ 4.6 {\rm K}$

 $f_{geo} = 8.82 \text{ GHz} \text{ (for single SQUID)}$ $f_{max} = 21.45 \text{ GHz} \text{ (for single SQUID)}$ Right: SNAP161A

12×12×2 rf SQUID array

with the same SQUIDs as the one-layer sample



3D rf SQUID Metamaterial Transmission Measurement

The passing microwave signal provides the rf flux Φ_{rf}



Measured quantity: Transmission S_{21} vs f

Variables: dc magnetic flux Φ_{dc} rf magnetic flux amplitude Φ_{rf} Frequency of rf flux, f Temperature, T





At low rf-flux we see the top of the tuning curve Asymmetric and hysteretic response with dc flux Simulations show loss of coherence on lower frequency branches

With increasing rf-flux

Tuning curve pushed down toward $f_{geo} = 8.82 \text{ GHz}$ Increasingly symmetric and less hysteretic response with dc flux

Two-layer 12×12×2 SNAP161A rf Power Dependence

Something qualitatively new happens!



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Laser scanning microscope combined with dilution fridge





A. P. Zhuravel, Verkin ILTPE Kharkiv, Ukraine

Low Temperature Physics

ARTICLE scitation.org/journal/ltp

Phase-resolved visualization of radio-frequency standing waves in superconducting spiral resonator for metamaterial applications



A. A. Leha,¹ A. P. Zhuravel,^{1,a)} A. Karpov,² A. V. Lukashenko,³ and A. V. Ustinov^{2,3}





Nb spiral at 4.5 K second harmonic $(f_2 = 220 \text{ MHz})$

Extend to the 1-20 GHz range and apply to rf SQUID metamaterials



Question for the Audience

Can any fabrication process in the world create Josephson junctions on completely independent layers?





Conclusions

- Macroscopic Quantum rf SQUID meta-atoms and metamaterials show transparency, bistability, intermodulation, strongly nonlinear response
- Coherence of rf SQUID metamaterials is enhanced by strong coupling and nonlinearity
- Imaging "dark modes" and the suppression of disorder to recover coherent response
- Next-generation 3D rf SQUID metamaterials have a large number of modes!
- rf SQUID metamaterials are a rich nonlinear medium

Thanks for your attention!



anlage@umd.edu

http://anlage.umd.edu



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In Memoriam Alexander P. Zhuravel Verkin Institute of Low Temperature Physics and Engineering Kharkiv, Ukraine (1953-2023)

1998 NATO Collaborative Linkage grant



The Co-PIs, Prof. Steven Anlage of the University of Maryland (left) and Dr. Alexander Zhuravel' of the Verkin Institute of Low Temperature Physics and Engineering, Kharkov, Ukraine (right). The picture is taken in Prof. Anlage's laboratory at the University of Maryland, Center for Superconductivity Research, and Physics Department. The scanning laser microscope is to the right of Dr. Zhuravel', while the equipment racks are behind him. Spring 2006 Erlangen, Germany

