Effects of Strong Capacitive and Inductive Coupling on Hysteretic rf SQUID Metamaterials

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Outline

• Brief Review of Superconducting Metamaterials

• rf SQUID Metamaterials
  • Tuning
  • Nonlinearity

• Collective Behavior of rf SQUID Metamaterials
  • Long-range Inductive Coupling
  • Laser Scanning Microscopy

• 3D Stacked rf SQUID Metamaterials
  • The Role of Capacitive Coupling

• Current / Future Work

• Conclusions
Motivation and Background

Metamaterials:
Artificial Structures made up of “meta-atoms” with new or extreme properties
Collections of sub-wavelength scatterers that create an effective medium with new functionality
Enhance light/matter interactions

Functional Metasurfaces

Extreme Nonlinearity
Intermodulation in a superconducting metamaterial


https://www.photonics.com/Articles/Metasurfaces_Open_Up_Research_Paths_For_Quantum/a68324
Why Superconducting Metamaterials?

Many exciting applications of metamaterials:
- Metasurface optics
- Cloaking
- Super-resolution imaging, etc. …

… have strict REQUIREMENTS on the metamaterials:
- **Ultra-Low Losses**
- Ability to scale down in size (e.g. $\lambda/10^2$) and texture the “meta-atoms”
- **Nonlinearity with wide and fast tunability of the index of refraction $n$**

… and superconductors bring these new features to the metamaterials field:
- Strong diamagnetism
- **Flux quantization and Josephson effects**
- Quantized energy states and quantum interactions with light

Negative Index Superconducting Metamaterials

How to make them: Step 1

All-Nb X-band waveguide + couplers

Nb X-band waveguide
(22.86 x 10.16 mm²)
$T_c = 9.25$ K

Nb Wires
0.25 mm dia.
$T_c = 9.25$ K

4.57 mm

10.2 mm

22.9 mm

Thanks to
P. Kneisel
@ Jefferson Lab

Negative Index Superconducting Metamaterials

How to make them: Step 2

Nb film, ~200 nm thick

0.89 cm

3.0 cm

Nb SRR
200 nm thick
on Quartz (350 µm)

$T_c = 8.65$ K

Negative Index Passband in a Superconducting Metamaterial
216 Split Ring Resonators in a 12-cell wire array, 9 cm long

Microwave Vector Network Analyzer: $S_{21}(f)$

Metamaterial

Waveguide

Cryogenic Environment

|$S_{21}$| (dB)
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Frequency (GHz)</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>13</td>
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</tbody>
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$\alpha = 5.08 \text{mm}$

NIR

SC

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SQUID = Superconducting QUantum Interference Device
A self-resonant meta-atom with very nonlinear properties

Resonant Frequency of rf SQUID

**rf SQUID Meta-Atoms**

A ‘Macroscopic Quantum’ Split-Ring Resonator

A self-resonant meta-atom with very nonlinear properties

**rf SQUID**

\[ \Phi \]

\[ \text{Nb: } T_c = 9.2K \]

\[ \text{Niobium Layer 2} \]

\[ \text{sc loop} \]

\[ \text{Junction} \]

\[ \text{Niobium Layer 1} \]

\[ \text{Via (Nb)} \]

\[ \text{Nb/AlO}_x/\text{Al/Nb} \]

\[ \Phi \]

\[ \text{Overlap} \to \text{forms capacitor} \]

\[ L_{geo} \]

\[ L_{JJ} \]

\[ R \]

\[ C \]
Josepthon Inductance is **large, tunable and nonlinear**

$$\delta \approx L_{JJ}$$

Resistively and Capacitively Shunted Junction (RCSJ) Model

$$L_{JJ} = \frac{\Phi_0}{2\pi I_c \cos(\delta)}$$

The “third Josephson effect”

When the JJ is incorporated into a loop and flux $\Phi$ is applied

Combines the Josephson effects with flux quantization

Superconducting Loop

External magnetic field essentially acts as a surrogate for the gauge-invariant phase
rf SQUID Superconducting Metamaterial

\[ \lambda \gg r, d \]

- Low loss
- Small Size
  - \( \lambda \sim 3 \text{ cm (} \sim 10 \text{ GHz) } \)
  - \( 2r = 20 \sim 800 \mu \text{m} \)

The SQUIDs interact by means of dipole – dipole coupling
\[ \rightarrow \] Collective Behavior

Original theory proposals:
N. Lazarides and G. P. Tsironis, APL 90, 163501 (2007)
The propagating EM wave provides the rf flux bias $\Phi_{rf}$.

The sample is free-standing, with no electrical contacts!

The wavelength $\lambda$ divided by the thickness $d$ is approximately $200$: $\lambda / d \approx 200$.
DC magnetic flux tuned resonance

Tunable Notch Filter

11x11 array, 4.4 K, -70 dBm

|\(S_{21}\) (dB)

\[\Phi_{dc} / \Phi_0\]

K-band cutoff

See similar work by P. Jung, et al., Appl. Phys. Lett. 102, 062601 (2013)
Applications of **Nonlinear** SQUID Metamaterials

Quantum-Limited Amplifiers

*A near-quantum-limited Josephson traveling-wave parametric amplifier*


*A wideband, low-noise superconducting amplifier with high dynamic range*

Byeong Ho Eom, Peter K. Day, Henry G. LeDuc and Jonas Zmuidzinas

*Two-tone spectroscopy of a SQUID metamaterial in the nonlinear regime*

E. I. Kiselev, A. S. Averkin, M. V. Fistul, V. P. Koshelets, and A. V. Ustinov

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Long-Range Inductive Coupling in rf SQUID metamaterials

Side-by-side SQUIDs have mutual inductance $M$ that falls off like a dipole-dipole interaction:

$$M < 0, \text{ with } |M| \sim \frac{1}{r^3}$$

Sample edges are also a large perturbation

**Chimera**: coexistence of synchronous and asynchronous groups of oscillations in a material, even for uniform constituent atoms and symmetric couplings

---

**Other theory papers motivated by our rf SQUID metamaterials**:

- N. Lazarides, Chaos, Solitons and Fractals 130, 109413 (2020).
Coupling Between SQUIDs Introduces Magneto-Inductive Modes

Simulation results for strong coupling (21 x 21 array), low power (linear), uniform array

Kuramoto coherence

Most Modes are “Dark”!

\[ r_A = \left| \frac{\sum_j^N A_j e^{i\theta_j}}{\sum_j^N A_j} \right| \]

 frequency (GHz)

0 0.3 0.6 1.0 1.3 1.4 1.415

Lazarides, arXiv:1310.5445

\( \delta_j(t) \)

0 2.5

0.6 -2.5

-0.6 -0.3

0.3 0

-0.3 0
Image rf SQUID Collective Response

Laser Scanning Microscope Imaging

Variables:
- Temperature
- dc magnetic flux
- rf magnetic flux
- rf frequency

Imaging experiments done by: A. P. Zhuravel in laboratory of A. Ustinov, KIT, Germany
Also Seokjin Bae and Jingnan Cai at UMD
Origins of rf Photoresponse

Contrast mechanisms

Heating of JJ leads to decrease of critical current and shift of resonance

\[ L_{JJ} = \frac{\Phi_0}{2\pi I_c(T) \cos \delta} \]

Photoresponse \( \sim |I_{rf}|^2 \) in junction
Imaging rf SQUID Metamaterial High-Power Coherent Mode

27 x 27 rf SQUID metamaterial

Photoresponse \sim |I_{rf}|^2

PhotoResponse Imaging of rf SQUID Metamaterial

Simulation Results
High Power (-20 dBm)
Coherent Mode

A. P. Zhuravel, P. Jung, S. Anlage, A. Ustinov, KIT, Germany
Imaging rf SQUID Metamaterial

The “Dark Modes” are now visible!

A weak global driving field reveals strong disorder of the sample

\[ T = 4.8 \text{ K} \]
\[ \Phi_{dc} = 0 \Phi_0 \]
\[ \Phi_{rf} = 10^{-4} \Phi_0 \]

27 x 27 array rf SQUID (12, 14)

Stronger global rf flux creates a coherent mode

\[ \Phi_{rf} = 10^{-3} \Phi_0 \]

Images taken at low rf flux amplitude

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Next-Generation rf SQUID Metamaterials

Move into the third dimension

Positive (Ferromagnetic) Coupling between meta-atoms

\[(M/L \rightarrow -0.2 \text{ to } 0 \text{ to } +0.70 \text{ as overlap increases})\]

Study 3D “stacked metamaterials”

A. M. Zagoskin J. Optics 14, 114011 (2012)

J. Shena, et al., Chaos 30, 123127 (2020)
Next-Generation rf SQUID Metamaterials
Move into the third dimension

2D metamaterial with mixed ferromagnetic and anti-ferromagnetic coupling

Thanks to Robin Cantor
@StarCryoelectronics
Josephson Junction Fabrication is Highly Constrained!

All JJs are fabricated on the base (orange) layer

The blue layer wiring is on top of the orange layer wiring

Note that large capacitors are formed where the blue layer wiring crosses the orange layer wiring

\[ C_{ov} \]

Note that dc currents only flow through the blue or orange loops

\[ C_{ov} \approx C, \text{ so the metamaterial dynamics are strongly altered!} \]
Overlapping SQUID geometry
Focus on a Corner-Coupled SQUID Pair

The JJs can only be fabricated on the base layer
Single SQUID Equations for $\delta(t)$
Flux Quantization vs. Faraday’s Law Approach

rf SQUID equation utilizing flux quantization and RCSJ model for current

$$\Phi = \Phi_{app} + \Phi_{ind}$$

Total flux quantization:

$$2\pi n = \delta + \frac{2e}{h} \Phi,$$

Combine and use the RCSJ model for $\Phi_{ind} = LI$:

$$\Phi_{dc} + \Phi_{rf} \sin \omega t = \frac{\Phi_0}{2\pi} \delta + L \left( I_c \sin \delta + \frac{\Phi_0}{2\pi} \frac{\delta}{R} + C \frac{\Phi_0}{2\pi} \frac{\delta}{\dot{\delta}} \right)$$

Second-order nonlinear differential equation for $\delta(t)$

Voltage on the junction:

$$V = \frac{\Phi_0}{2\pi} \dot{\delta}$$

rf SQUID equation utilizing Faraday’s law

$$V = \frac{d}{dt} \left( \Phi_{app} + \Phi_{ind} \right)$$

Faraday’s law applied to a single SQUID results in the time-derivative of the rf SQUID equation

$$\frac{d}{dt} \left[ \Phi_{dc} + \Phi_{rf} \sin \omega t \right] = \frac{d}{dt} \left[ \frac{\Phi_0}{2\pi} \delta + L \left( I_c \sin \delta + \frac{\Phi_0}{2\pi} \frac{\delta}{R} + C \frac{\Phi_0}{2\pi} \frac{\delta}{\dot{\delta}} \right) \right]$$
Two Corner-Coupled SQUID Equations

Faraday’s Law / Voltage Approach

Now there are voltage drops in the SQUID loops that are not simply related to $\dot{\delta}$

2-SQUID flux relation:

$$\left( \Phi_{a}^{app} \Phi_{b}^{app} \right) = \frac{\Phi_{0}}{2\pi} \left( \delta_{a} \delta_{b} \right) + \left( \Phi_{a}^{ind} \Phi_{b}^{ind} \right)$$

Apply Faraday’s law to the central loop:

$$V_{b} - V_{1} + V_{2} = \frac{d}{dt} \left( \Phi_{cen}^{app} - \Phi_{cen}^{ind} \right)$$

where $V_{1}$ and $V_{2}$ are the voltages across the capacitors, $V_{b}$ is the voltage across junction $b$

Conservation of current through the overlap capacitors:

$$I_{a0} + I_{b0} = I_{a1} + I_{b1}$$

$$\dot{V}_{1} = -\dot{V}_{2}$$

The flux relations become:

$$\left( \Phi_{dc} + \Phi_{rf} \sin(\omega t) \right) = \frac{\Phi_{0}}{2\pi} \left( \delta_{a} \delta_{b} \right) + \left( L_{geo} M \right) \begin{pmatrix} I_{a0} \\ I_{b0} \end{pmatrix}$$

$$C_{ov} \begin{pmatrix} -L_{\delta a} \dot{V}_{1} \\ L_{\delta b} \dot{V}_{1} \end{pmatrix}$$

with

$$\dot{V}_{1} = [(1 + \kappa_{eb}) \dot{V}_{b} + \kappa_{va} \dot{V}_{a} + L_{j_{b}} \dot{I}_{b} + L_{j_{a}} \dot{I}_{a}]$$

$$-\dot{\Phi}_{cen}^{app} - \kappa_{va} \dot{\Phi}_{a}^{app} - \kappa_{vb} \dot{\Phi}_{b}^{app}] / 2$$

and $L_{\delta a}, L_{\delta b}$ are partial inductances

This is a pair of coupled 4th-order nonlinear equations for $\delta_{a}, \delta_{b}$.

These can be reduced to a set of 6 first-order equations (utilizing constraints)

Write $\delta = \delta_{dc} + \delta_{rf} e^{i\omega t}$, and linearize by assuming $|\delta_{rf}| \ll 1$, results in a characteristic equation for $\omega$ of 6th order, yielding 3 positive and 3 negative frequency solutions.
Resonant Modes of Two Corner-Coupled rf SQUIDs

\[ \Omega = \frac{\omega}{\omega_{geo}} \]

Red line: linearized solutions

Dark features: full nonlinear solution in the $\Phi_{rf} \ll 1$ limit

NO flux quantization in this loop

single-SQUID tuning curve

$\beta_{rf} = \frac{L_{geo}}{L_{JJ}} = 5.48$

$f_{geo} = 8.36 \text{ GHz}$
Resonant Modes of Four Corner-Coupled rf SQUIDs

\[ \Omega = \frac{\omega}{\omega_{geo}} \]

linearized
eigenmodes

extra-SQUID
mode (1)

partial-SQUID
modes (4)

full-SQUID
modes (4)

https://doi.org/10.48550/arXiv.2402.07044
Resonant Modes of $N \times N \times 2$ Corner-Coupled rf SQUIDs

$2N^2$ SQUID loops

$(2N - 1)^2$ partial loops

$2(N - 1)^2$ extra SQUID loops

Total: $8N^2 - 8N + 3$

Gauge-invariant phase differences $\delta$: $2N^2$

Each loop contributes one equation, giving a total number of equations: $8N^2 - 8N + 3$

There are a total of $8N^2 - 8N + 3$ unknowns, resulting in a total number of modes: $8N^2 - 8N + 3$

$N = 2$ case
19 modes

$N = 12$ case
1,059 modes
Next-Generation rf SQUID Metamaterials
Compare 2D and 3D samples made up of the same rf SQUIDs

Left: SNAP161D
12×12×1 rf SQUID array

\[ \beta_{rf} = 5.48 \text{ } \text{at } 4.6K \]

\[ f_{geo} = 8.82 \text{ GHz (for single SQUID)} \]

\[ f_{max} = 21.45 \text{ GHz (for single SQUID)} \]

Right: SNAP161A
12×12×2 rf SQUID array
with the same SQUIDs as the one-layer sample
3D rf SQUID Metamaterial Transmission Measurement

The passing microwave signal provides the rf flux $\Phi_{rf}$

Measured quantity: Transmission $S_{21}$ vs $f$

Variables:
- dc magnetic flux $\Phi_{dc}$
- rf magnetic flux amplitude $\Phi_{rf}$
- Frequency of rf flux, $f$
- Temperature, $T$
Single-layer SNAP161D rf Power Dependence

At low rf-flux we see the top of the tuning curve
Asymmetric and hysteretic response with dc flux
Simulations show loss of coherence on lower frequency branches

With increasing rf-flux
Tuning curve pushed down toward $f_{\text{geo}} = 8.82 \text{ GHz}$
Increasingly symmetric and less hysteretic response with dc flux

$\beta_{\text{rf}} = 5.48 @ 4.6K$
Two-layer $12\times12\times2$ SNAP161A rf Power Dependence

Something qualitatively new happens!

$N = 12$ case
1.059 modes

The low-frequency bands show $1-\Phi_0$ tuning
Higher frequency bands show $0.93-\Phi_0$ tuning

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Laser scanning microscope combined with dilution fridge

Data collecting computer

Microwave source

Lock-in1 photoresponse

Lock-in2 Reflectivity

Microwave detector

photo diode

4f Scanning System

Laser Source

modulation

microwave

Laser light

Cold finger

4f Scanning System

Dilution Refrigerator from BlueFors

cold finger & sample

Thanks to A. P. Zhuravel for assistance with the optical design

Phase-Sensitive LSM Imaging  
A. P. Zhuravel, Verkin ILTPE Kharkiv, Ukraine

Phase-resolved visualization of radio-frequency standing waves in superconducting spiral resonator for metamaterial applications

Extend to the 1-20 GHz range and apply to rf SQUID metamaterials
Question for the Audience

Can any fabrication process in the world create Josephson junctions on completely independent layers?
Conclusions

• Macroscopic Quantum rf SQUID meta-atoms and metamaterials show transparency, bistability, intermodulation, strongly nonlinear response

• Coherence of rf SQUID metamaterials is enhanced by strong coupling and nonlinearity

• Imaging “dark modes” and the suppression of disorder to recover coherent response

• Next-generation 3D rf SQUID metamaterials have a large number of modes!

• rf SQUID metamaterials are a rich nonlinear medium

Thanks for your attention!

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In Memoriam
Alexander P. Zhuravel
Verkin Institute of Low Temperature Physics and Engineering
Kharkiv, Ukraine
(1953-2023)

1998 NATO Collaborative Linkage grant

Spring 2006 Erlangen, Germany

The Co-PIs, Prof. Steven Anlage of the University of Maryland (left) and Dr. Alexander Zhuravel of the Verkin Institute of Low Temperature Physics and Engineering, Kharkiv, Ukraine (right). The picture is taken in Prof. Anlage’s laboratory at the University of Maryland, Center for Superconductivity Research, and Physics Department. The scanning laser microscope is to the right of Dr. Zhuravel, while the equipment racks are behind him.