

ULTIMATE TEMPERATURE OF PULSE TUBE CRYOCOOLERS

P. Kittel

Consultant
Palo Alto, CA 94303, USA

ABSTRACT

An ideal pulse tube cryocooler using an ideal gas can operate at any temperature. This is not true for real gasses. The enthalpy flow resulting from the real gas effects of ^3He , ^4He , and their mixtures in ideal pulse tube cryocoolers puts limits on the operating temperature of pulse tube cryocoolers. The discussion of these effects follows a previous description of the real gas effects in ideal pulse tube cryocoolers and makes use of models of the thermophysical properties of ^3He and ^4He . Published data is used to extend the analysis to mixtures of ^3He and ^4He .

The analysis was done for pressures below 2 MPa and temperatures below 2.5 K. Both gasses and their mixtures show low temperature limits for pulse tube cryocoolers. These limits are in the 0.5-2.2 K range and depend on pressure and mixture. In some circumstances, even lower temperatures may be possible. Pulse tube cryocoolers using the two-fluid properties of dilute ^3He in superfluid ^4He appear to have no limit.

KEYWORDS: Cryocoolers, Real gas effects, Pulse tube theory

INTRODUCTION

An ideal pulse tube cryocooler using an ideal gas has a cooling power given by [1]

$$Q_c/W = T_c/T_h \quad (1)$$

Such a cooler does not have a minimum operating temperature. It is capable of cooling to any temperature. When real gasses are used, the performance of pulse tube cryocoolers is affected at all temperatures [2-6]. The performance is significantly reduced for temperatures near and below the critical temperature of the working fluid. This was shown analytically for ^4He [2-3] and ^3He [5]. Since the critical point of ^3He occurs at a lower temperature than for ^4He , pulse tube coolers using ^3He reach lower temperatures than those using ^4He . This has been demonstrated in practice [3-4]. The existence of low temperature limits of pulse tube cryocoolers was introduced in a recent study [7].

This paper reports the results of an analysis of the ultimate operating temperatures for an ideal pulse tube cryocooler for pressures below 2 MPa using ^3He , ^4He , and their mixtures as the working fluid in ideal pulse tube cryocoolers.

APPROACH

The approach has been previously described for ideal and non-ideal pulse tubes [8-9], for ^4He real gas effects in ideal pulse tube cryocoolers [1], and for ^3He real gas effects in ideal pulse tube cryocoolers [5].

We make the following assumptions. Pressure gradients that result from viscous flows in the orifice or inertance tube are proportional to and in phase with the mass flow. Similarly, heat transfer is proportional to and in phase with temperature differences. All oscillating quantities are sinusoidal at the fundamental frequency. These are the assumptions of small amplitudes and that the energy carried by higher harmonics is small compared to the total energy carried by the system and can be ignored.

The approach assumes that the system (FIGURE 1) is ideal except for the gas properties, which are the real gas properties. The gas properties for pure ^3He and ^4He were taken from thermophysical models [10]. In ideal, lossless components, there is no entropy generation. Thus, the effects of viscous flow losses (except in the orifice or inertance tube), secondary flows, and inadequate regenerator heat capacity are ignored. Heat flows (thermal conduction) are treated as an external load. Only steady state operation is considered. The system control volume is a closed system (no mass flows in or out).

In steady state operation the mean enthalpy and entropy flows are given by

$$\dot{X} = \frac{1}{\tau} \int_0^{\tau} \dot{m} x dt \quad (2)$$

where τ is the period of oscillation, x is h or s and X is H or S , respectively.

IDEAL GAS PROPERTIES

If the working fluid is assumed to be an ideal gas, then

$$ds = c_p dT/T - R dP/P \quad (3)$$

and

$$dh = c_p dT \quad (4)$$

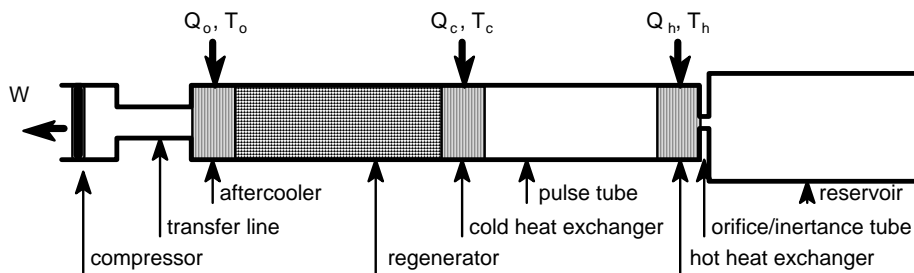


FIGURE 1. The principal components of an Orifice Pulse Tube Cryocooler showing the location and nominal direction of the heat and work for a control volume enclosing the entire cryocooler.

where $c_p = 5/2 R$ for helium.

In the pulse tube, an adiabatic region, $ds = 0$, thus equations (3-4) become

$$dT/dP = V/c_p \quad (5)$$

and
$$dh = V dP \quad (6)$$

where the coefficient of dP in equation (6) is evaluated at the temperature of the cold heat exchanger. Elsewhere in the pulse tube, VP increases with temperature. However, because no work is done and the pulse tube is adiabatic, there is no change in the enthalpy flow within the pulse tube. To maintain constant enthalpy flow, the mass flow decreases with increasing temperature. This accumulation effect was discussed earlier for ideal gas flows [9]. In adiabatic sections the temperature and pressure oscillations are in phase.

In the regenerator, there are no temperature oscillations, $dT = 0$, thus equations (3-4) become

$$ds = (V/T) dP \quad (7)$$

and
$$dh = 0. \quad (8)$$

Thus there is no enthalpy flow in the regenerator.

REAL GAS PROPERTIES

When real gas effects are included, equations (3-4) become [11]

$$T ds = c_p dT - T\beta V dP \quad (9)$$

and
$$dh = c_p dT + [1 - T\beta]V dP \quad (10)$$

where $\beta = 1/V dV/dT|_P$ is the volume expansivity. Also, c_p is no longer $5/2 R$, but is a function of T and P . For an ideal gas, $T\beta = 1$.

Pulse Tube

Within the pulse tube, the flow is adiabatic. Thus, $ds = 0$ and equation (9-10) become

$$dT/dP = T\beta V/c_p \quad (11)$$

and
$$dh = V dP. \quad (12)$$

where, as in equation (6), the coefficient of dP in equation (12) is evaluated at the temperature of the cold heat exchanger.

Regenerator

In regenerators, heat transfer to the regenerator material suppresses the gas temperature amplitude. Thus, $dT = 0$. For a real gas, equations (9-10) become:

$$ds = -V\beta dP \quad (13)$$

and
$$dh = [1 - T\beta]VdP. \quad (14)$$

DISCUSSION

The heat absorbed at the cold heat exchanger is given by the difference at the cold heat exchanger between the enthalpy flows in the regenerator and in the pulse tube. For an ideal gas, this is the difference between equations (6) and (8):

$$\Delta dh = V dP. \quad (15)$$

where equations (6) and (8) have been combined.

For a real gas, the heat absorbed is the difference between equations (12) and (14):

$$\Delta dh = T\beta V dP \quad (16)$$

Pure ^3He and ^4He Fluids

Equation (15) places no limits on the temperature at which an ideal pulse tube cooler can operate using an ideal gas. Equation (16) does place temperature limits on an ideal pulse tube cooler using a real gas. The cooling reaches a limit when $\beta = 0$ and cooling ceases. The locus of $\beta = 0$ are shown in FIGURE 2. The ^3He curve and the ^4He curve near 2.2 K were calculated from reference [10]. The lower ^4He curve was calculated using reference [12]. For ^4He , $\beta < 0$ between the two lines marked ^4He and $\beta > 0$ elsewhere. For ^3He , $\beta < 0$ for temperatures below the line and $\beta > 0$ for higher temperatures.

In the regions where $\beta > 0$, pulse tube coolers will operate in the usual fashion with the mass flow and pressure oscillations close to being in phase [13]. For coolers starting near room temperature, a cooler could reach ≈ 2.2 K using ^4He and ≈ 1 K using ^3He .

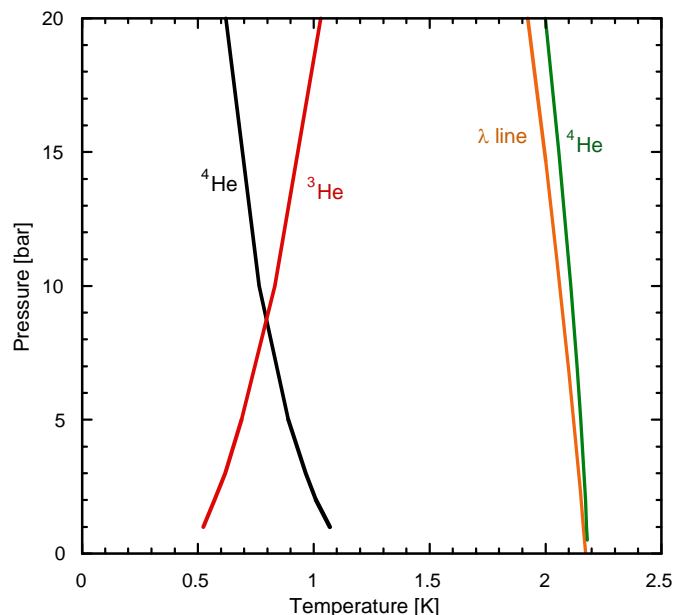


FIGURE 2. The locus in pressure-temperature space of where the cooling power of an ideal pulse tube cryocooler is zero for ^3He and ^4He working fluids. Also shown is the lambda-line of ^4He .

Below ≈ 1 K, the ^4He β is again > 0 . A cooler operating in this region would need to reject heat to a pulse tube cooler using ^3He at ≈ 1 K. Unfortunately, the ^4He β is very small below 1 K (see FIGURE 3), making it difficult to realize a practical cooler in this region.

In the regions where $\beta < 0$, a cooler using the usual phase shift between mass flow and pressure would produce negative cooling (heating). Perhaps one could reverse the roles of the pulse tube heat exchangers to make a cooler; or perhaps one could change the phase between mass flow and pressure by an additional 180° . In practice, neither approach is likely to work. The ^3He β is quite small below 1 K (see FIGURE 3) and ^4He has a high thermal conductivity below the lambda-transition that will thermally short the pulse tube.

Mixed ^3He and ^4He Fluids

Mixtures of ^3He and ^4He offer the possibility of adjusting the gas properties to modify performance. While the thermodynamic state of a pure fluid can be specified by two parameters such as T and P, a two-component mixture requires additional parameters, x_3 , the molar fraction of ^3He in the mixture and the chemical potential, μ , of each isotope. In steady state operation, a pulse tube cooler will have concentration gradients caused by the two isotopes having different viscosities and by chemical potential effects. The ideal pulse tube cooler model ignores viscous effects except for the pressure drop in the orifice. We will assume that above the lambda-line that the concentration gradients do not significantly contribute to the ultimate temperature and the ultimate temperature is determined only by the concentration at the cold heat exchanger. I.e., we assume that the loci of $dV/dT|_{P,\mu} = 0$ and $dV/dT|_{P,x_3} = 0$ are the same.

There is only limited data on the volume expansivity of helium mixtures in the 1-2 K and < 2 MPa temperature and pressure range. In FIGURE 4, $\beta = 0$ data are shown as a function of mixture ratio superimposed on the $^3\text{He}/^4\text{He}$ phase diagram. All of the data are for saturated vapor pressure rather than constant pressure. The mixture data is for constant x_3 .

The pure fluid data points for the upper branch of $\beta = 0$ ($x_3 = 0$ and $x_3 = 1$) were calculated [10]. The other two data points are interpolated from a table [14]. In addition to errors in the published data, the accuracy of the interpolation is about ± 40 mK because of

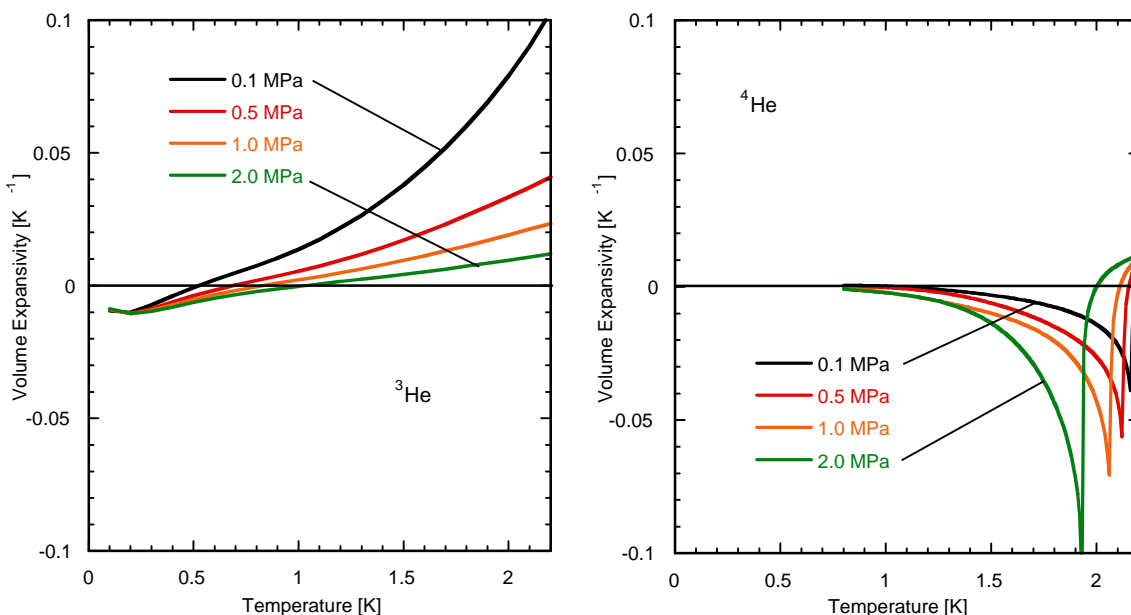


FIGURE 3. Volume expansivity of ^3He (left) and ^4He (right).

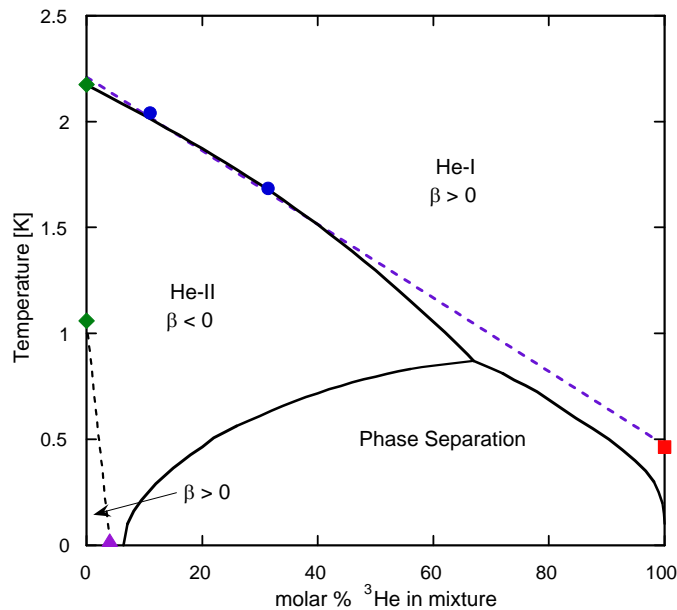


FIGURE 4. The $\beta = 0$ data for ${}^3\text{He}/{}^4\text{He}$ mixtures at saturated vapor pressure superimposed on the ${}^3\text{He}/{}^4\text{He}$ phase diagram. The data are discussed in the text.

the large values of the second and higher order derivatives near $\beta = 0$. These four data points are well fit by a linear function of x_3 , thus, further justifying the assumption of non-interacting components. All the upper branch of $\beta = 0$ lies in the He-I region.

The pure fluid data point for the lower branch of $\beta = 0$ ($x_3 = 0$) was calculated [12]. The other data point is an upper limit estimated from data [15]. The actual $\beta > 0$ region is likely smaller than indicated.

${}^3\text{He}$ Quasi Particles in Superfluid ${}^4\text{He}$

The proceeding discussion assumes that the mixture moves as a bulk fluid. In the superfluid region, He-II, the fluid can act as two interpenetrating fluids, which can move independently. One is the pure ${}^4\text{He}$ superfluid component. The other is the normal fluid component consisting of some ${}^4\text{He}$ and all of the ${}^3\text{He}$. Near the λ -line, most of the fluid is in the normal state. Below ≈ 1 K, the normal fluid consists of only ${}^3\text{He}$ quasi particles. If the piston in the compressor has a superleak then the ${}^3\text{He}$ can act as a low-density gas moving in a fixed ${}^4\text{He}$ background. A full treatment of the thermodynamics in this regime is beyond the scope of this paper. Reviews of the thermodynamics of dilute ${}^3\text{He}$ in ${}^4\text{He}$ solutions have been given elsewhere [16-18].

A pulse tube using dilute ${}^3\text{He}$ in a fixed ${}^4\text{He}$ background was developed at Los Alamos. It was filled with $x_3 = 0.17$ and operated between 1 K and 0.6 K [19]. The system pressure is fixed by the conditions in the compressor/aftercooler. The compressor does not cause pressure oscillations; rather it causes the ${}^3\text{He}$ concentration, x_3 , and the osmotic pressure, Π_3 , to oscillate. Pressure gradients in the regenerator, heat exchangers, and orifices are replaced by gradients in Π_3 . In the pulse tube, the constraint that $\nabla P = 0$ is replaced by $\nabla \mu_4 = 0$, where μ_4 is the chemical potential of the ${}^4\text{He}$. Several loci of $\nabla \mu_4 = 0$ (from data in reference [16]) are shown in FIGURE 5 from which one sees that in the pulse tube x_3 increases as the temperature decreases. The cold end temperature is limited by phase separation. To reach the lowest temperatures limits $x_3 \leq 0.064$. Such a pulse tube with a warm end above 0.6 K will have $x_3 < 0.01$ at the warm end and in the compressor. Such low concentrations limits the mass flow and the cooling achievable in a practical

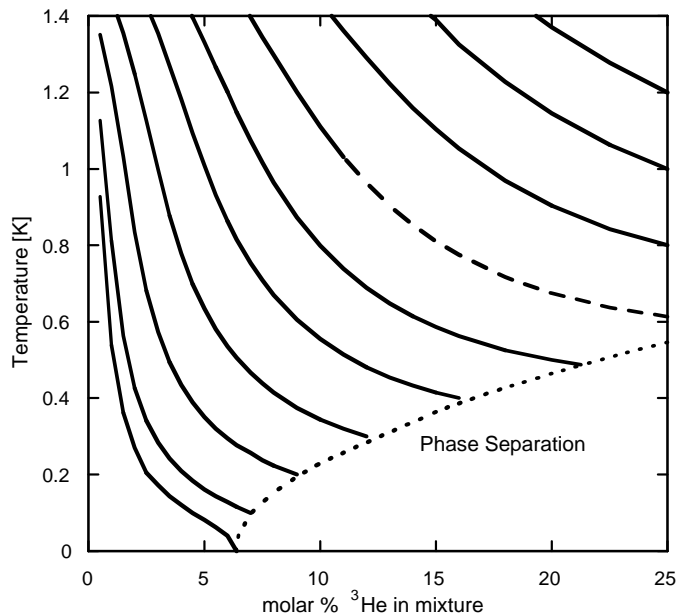


FIGURE 5. Selected loci of $\nabla\mu_4 = 0$ for dilute $^3\text{He}/^4\text{He}$ mixtures. The dashed line is, approximately, the operating region of the Los Alamos pulse tube.

sized cooler. This type of cooler has not seen further development nor has it seen any practical application

The present thermodynamic model of dilute ^3He in ^4He [16-18] shows no evidence of a low temperature limit of a pulse tube cryocooler.

SUMMARY

The theoretical limit of a non-superfluid pulse tube cryocooler is when the volume expansivity is zero: $\beta = 0$. Below ≈ 1 K, ^3He concentration driven pulse tube cryocoolers are possible with no known ultimate limiting temperature.

In practice, a conventional pulse tube cryocooler using ^4He is limited to temperatures above ≈ 2.2 K. Using ^3He , the limit is above ≈ 1 K. The limit can be adjusted in between these two points by using a mixture of ^4He and ^3He . When mixtures are used, the mixture ratio will not be constant throughout the cooler. The mixture ratio will vary with position because the chemical potential of the mixture depends on temperature and pressure. The mixture at the cold heat exchanger will not be the same as the initial mixture; rather, the mixture stratifies. Thus, the ultimate achievable temperature of a cooler using a mixture will depend on the mixture ratio at the cold heat exchanger when the cooler is operating in steady state.

ACKNOWLEDGEMENTS

Dr. Vincent Arp of Cryodata and Dr. Yonghua Huang of Shanghai Jiao Tong University gave invaluable assistance with the thermodynamic properties of helium. Prof. A.T.A.M. de Waele of Eindhoven University of Technology for suggesting this topic and many interesting discussions.

This work was funded, in part, through University Affiliated Research Center (UARC) Subcontract S01817969. The UARC is managed by the University of California, Santa Cruz under NASA Ames Research Center Contract NAS2-03144.

NOMENCLATURE

Roman

H	Enthalpy	c	Specific heat
P	Pressure	h	Specific enthalpy
Q	Heat	m	Mass
R	Gas constant (per mole)	s	Specific entropy
S	Entropy	x	Molar fraction of ^3He in mixture
T	Temperature		
V	Specific volume		
W	Work		

Greek

β	Thermal expansivity
μ	Chemical potential
Π	Osmotic pressure

Subscripts

3	^3He component
c	Cold
h	Hot
P	Isobaric

REFERENCES

- Kittel, P., "Ideal Orifice Pulse Tube Refrigerator Performance," *Cryogenics* **32** pp. 843-844 (1992).
- Kittel, P., "Enthalpy, Entropy, and Exergy Flows – Real Gas Effects in Ideal Pulse Tube Cryocoolers," in *Advances in Cryogenic Engineering* 51, edited by J. Weisend et al., AIP Press, Melville, 2006, pp. 345-352.
- Xu, M.Y., de Waele, A.T.A.M., and Ju, Y.L., *Cryogenics* **39**, pp 865-869 (1999).
- Jiang, N., Lindemann, U., Giebeler, F. and Thummes, G., *Cryogenics* **44**, pp 809-816 (2004).
- Kittel, P., "Performance Limits of Pulse Tube Cryocoolers Using ^3He ," in *Advances in Cryogenic Engineering* 53, edited by J. Weisend et al., AIP Press, Melville, 2008, pp. 1421-1428.
- Kittel, P., "The Role of Thermoconductivity in Pulse Tube Cryocoolers," in *Cryocoolers* 15, edited by S.D. Miller and R.G. Ross, Jr., Springer, New York, 2009, pp. 281-288.
- de Waele, A.T.A.M., Xu, M.Y., and Jug, Y.L., *Physica B* **284-288**, pp. 2018-2019 (2000)
- Kittel, P., "Enthalpy, Entropy, and Exergy Flows in Ideal Pulse Tube Cryocoolers," in *Cryocoolers* 13, edited by R.G. Ross, Jr., Springer, New York, 2005, pp. 333-342.
- Kittel, P., "Enthalpy, Entropy, and Exergy Flow Losses in Pulse Tube Cryocoolers," in *Cryocoolers* 13, edited by R.G. Ross, Jr., Springer, New York, 2005, pp. 343-352.
- HE3PAK v1.2 and HEPAK v3.4, available from Horizon Technologies, Littleton, CO, USA.
- Black, W.Z. and Hartley, J.G., *Thermodynamics*, 3rd Edition, HarperCollins, New York, 1996, ch. 9.
- Arp, V., Preliminary version of HEPAK v4, private communication, Aug 2008.
- Radebaugh, R., Lewis, M., Luo, E., Pfothenhauer, J. M., Nellis, G. F., and Schunk, L. A., "Inertance Tube Optimization for Pulse Tube Refrigerators," in *Advances in Cryogenic Engineering* 51, edited by J. Weisend et al., AIP Press, Melville, 2006, pp. 59-67.
- Esel'son, B.N., Grigor'ev, V.N., Ivantsov, V.G., Rudavskii, E.Ya., Sanikadze, D.G., and Serbin, I.A., *Solutions of $\text{He}^3 - \text{He}^4$ Quantum Liquids*, Nauka, Moscow, 1973 (in Russian) p. 424.
- Ebner, C. and Edwards, D.O., *Physics Reports* **2**, pp. 77-154 (1971).
- Radebaugh, R., *Thermodynamic Properties of $\text{He}^3 - \text{He}^4$ Solutions with Applications to the $\text{He}^3 - \text{He}^4$ Dilution Refrigerator*, NBS TN 362 (1967), available from NTIS (www.ntis.gov).
- Kuerten, J.G.M. , Castelijns, C.A.M. , de Waale, A.T.A.M. , and Gijsman, H.M., *Cryogenics* **24**, pp. 419-443 (1985)
- Chaudhry, G. and Brisson, J.G., "Thermodynamic properties of $^3\text{He} - ^4\text{He}$ mixtures between 0.15 K and 1.8 K," *Cryo. Engin. Conf.*, Tucson, AZ (2009) and *J. Low Temp. Phys.* (submitted).
- Watanabe, A., Swift, G.W., and Brisson, J.G., "Superfluid Orifice Pulse Tube Refrigerator below 1 Kelvin," in *Advances in Cryogenic Engineering* 41, edited by P. Kittel, Plenum Press, New York, 1996, pp. 1519-1526.