

Theoretical simulation studies of pulsed field magnetisation of (RE)BCO bulk superconductors

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Abstract. The magnetisation of bulk high temperature superconductors (HTS), such as RE-Ba-Cu-O [(RE)BCO, where RE is a rare earth element or Y], by a practical technique is essential for their application in high field, permanent magnet-like devices. Research to-date into the pulsed field magnetisation (PFM) of these materials, however, has been limited generally to experimental techniques, with relatively little progress in the development of theoretical models. This is because not only is a multi-physics approach needed to take account of the heating of the samples but also the high electric fields generated are well above the regime in which there are reliable experimental results. This paper describes a framework of theoretical simulation using the finite element method (FEM) that is applicable to both single- and multi-pulse magnetisation processes of (RE)BCO bulk superconductors. The model incorporates the heat equation and provides a convenient way of determining the distribution of trapped field, current density and temperature change within a bulk superconductor at each stage of the magnetisation process. An example of the single-pulse magnetisation of a (RE)BCO bulk is described. Potentially, the model may serve as a cost-effective tool for the optimisation of the bulk geometry and the magnetisation profile in multi-pulse magnetisation processes.

1. Introduction

The magnetisation of bulk high temperature superconductors (HTS), such as RE-Ba-Cu-O [(RE)BCO] where RE represents a rare earth element or Y, by a practical technique is essential for their application in high field, permanent magnet-like devices. The pulsed field magnetisation (PFM) technique has been applied to (RE)BCO bulk superconductors since 1996 [1,2], and is considered more suitable and convenient for *in situ* magnetisation of these materials compared to conventional techniques of static field magnetisation such as ‘field cooling’ (FC) and ‘zero field cooling’ (ZFC). Several multi-pulse magnetisation techniques have been developed since the late 1990s [3-5], where repeated pulses of lower intensity are applied to bulk HTS mainly in the temperature range 30-60 K in order to suppress the heat generation associated with rapid flux motion in the samples. A trapped field of 5.20 T was achieved in a GdBCO bulk at 30 K in 2006 [6], which is the highest trapped field by the PFM processes reported to-date.

Existing research into the pulsed field magnetisation of bulk HTS has been limited generally to experimental techniques, with relatively little progress in the development of theoretical models. This is because not only is a multi-physics approach needed to take account of the heating of the samples,

but also the high electric fields generated during the magnetisation process are well above the regime in which there are reliable experimental results. This paper describes a framework of theoretical simulation using the finite element method (FEM) that is applicable to both single- and multi-pulse magnetisation processes of (RE)BCO bulk materials. The model incorporates the heat equation and provides a convenient way of determining the distribution of trapped field, current density and temperature change within a bulk superconductor at each stage of the magnetisation process.

2. Method

An approach developed by A. M. Campbell in 2007 [7], based on the force-displacement curve of magnetic flux in HTS [8,9], has been adopted in the present model. This approach solves directly the critical state, in which $J = J_c$ wherever magnetic flux penetrates the sample. A term regarding the viscous force, which could be significant in the PFM processes, is also included in Campbell's equation, as follows;

$$\nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J}_p + \mathbf{k} \left[1 - \exp \left(- \left| \frac{(A - A_p) \mu_0 J_c}{k A_r} \right| \right) \right] - \frac{\mu_0}{\rho_v} \frac{\partial \mathbf{A}}{\partial t} \quad (1)$$

where

$$\mathbf{k} = \mu_0 \mathbf{J}_c \text{sign}(A_p - A) - \mu_0 \mathbf{J}_p \quad (2)$$

Here A_p and \mathbf{J}_p are the magnetic vector potential and the corresponding current density of the initial state, respectively, A_r is regarded as a constant dependent on the material when a sufficiently 'large uniform' field is applied to the superconductor [7], and ρ_v is the flux flow resistivity of the (RE)BCO bulk material.

It is worth pointing out that the normal state resistivity of the material is assumed for ρ_v and that the model is not based on an E - J power law relationship since the electric field is extremely high. The electric field generated during a PFM process is typically $10^{-2} \text{ V cm}^{-1}$, which can be estimated using the following equation;

$$E = r \frac{\partial B}{\partial t} \quad (3)$$

where r is the radius of the bulk. The electric field involved when measuring the commonly used E - J power law is usually around $10^{-6} \text{ V cm}^{-1}$ at J_c [10], and a typical power law of J^{20} would imply a resistivity that is many orders of magnitude larger than the normal state resistivity which must be an upper limit. Thus the electric field generated in a PFM process is in a completely different regime from that in an E - J characterisation, which is beyond any reliable experimental results that are currently available. We therefore regard ρ_v as an adjustable parameter which can be determined from experimental results. The best estimate we can make at present is the normal state resistivity. One of the main objectives of this paper is to demonstrate that we can predict the magnetisation of a (RE)BCO bulk superconductor with a reasonable estimate of ρ_v .

The heat equation and the Kim model [11] are also incorporated in the model. These concepts are represented by equations (4) and (5), respectively;

$$K \nabla \cdot (\nabla T) = c \rho \frac{\partial T}{\partial t} - \left| J_c \frac{\partial A}{\partial t} \right| \quad (4)$$

$$J_c = \frac{J_{c0}}{1 + \frac{B}{B_0}} \quad (5)$$

In Eq. (4), K is the thermal conductivity of the bulk (RE)BCO material, c is the heat capacity, and ρ is the mass density. In Eq. (5), J_{c0} and B_0 are constants dependent on the superconductor. In the present

model, J_{c0} and B_0 are set as 4×10^4 A cm⁻² and 0.37 T respectively, which are typical values for bulk (RE)BCO at approximately 77 K.

The coordinate system is established in the cross section of the bulk superconductor, as shown schematically in Fig. 1(a). The origin O is set at the centre of the cross section, and the x - and y -axes are defined along the radius and the thickness of the sample, respectively. Simulations are conducted for the first quadrant only in view of the symmetry of the superconductor. A typical FEM grid generated in FlexPDE 6 is shown in Fig. 1(b). The example (RE)BCO bulk sample is 20 mm in diameter and 10 mm in thickness.

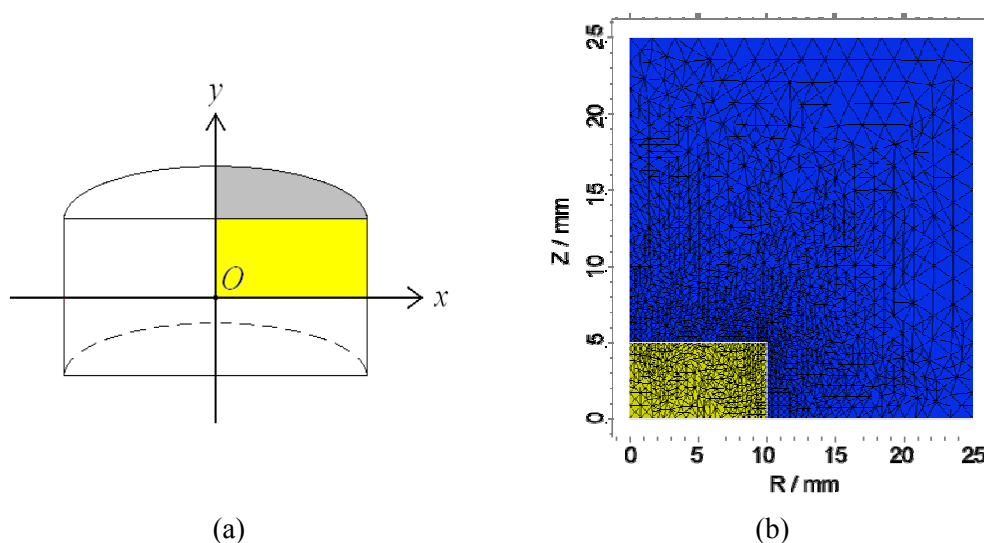


Figure 1. (a) Schematic illustration of the coordinate system established in the cross section of the (RE)BCO bulk superconductor. The first quadrant is marked in yellow. (b) A typical FEM grid of the first quadrant generated in FlexPDE 6. The radius (x -axis) and the thickness (y -axis) are represented by the R - and Z -axes, respectively. The cross section of the superconductor is shown in yellow and free space is shown in blue.

The present model involves dividing a pulse into two stages, the ascending stage and the descending stage, with a number of intermediate points set within each stage. Campbell's equation, the heat equation and the Kim model are solved simultaneously in FlexPDE, and the simulation results for each of these points are recorded for monitoring the evolution of the magnetisation process, including mainly the distributions of magnetic field lines, trapped field, current density, temperature change and heat flow within the superconductor. Simulations for multi-pulse processes can be realised simply by running the model repeatedly. For example, a pulse applied in the model could have a peak value of 5 T, with an ascending stage of 30 ms and a descending stage of 150 ms.

3. An example of simulation

As an example, we simulate in FlexPDE the magnetisation process of a (RE)BCO bulk sample by a single pulse at around 77 K. Fig. 2 illustrates the change in the distribution of magnetic field lines with the evolution of the magnetisation process. Fig. 2(a) and (b) shows that the flux lines enter the superconductor predominantly from the edge of the surfaces of the sample, i.e. the upper-right corner of the small rectangle in each part of the figure, when the pulse rises from 0 to 1.5 T. The superconductor is then almost fully penetrated when the external field reaches 2 T. Thereafter, little change in the shape of the flux lines is observed in the model until the pulse decreases to 2 T, although the magnitude of the magnetic field within the sample changes with the external field continuously, as shown typically in Fig. 2(c). A more significant change in the shape of the flux profile occurs during

the second half of the descending stage, before a final distribution similar to that of a cylindrical permanent magnet is achieved within the superconductor, as indicated in Fig. 2(d)-(f).

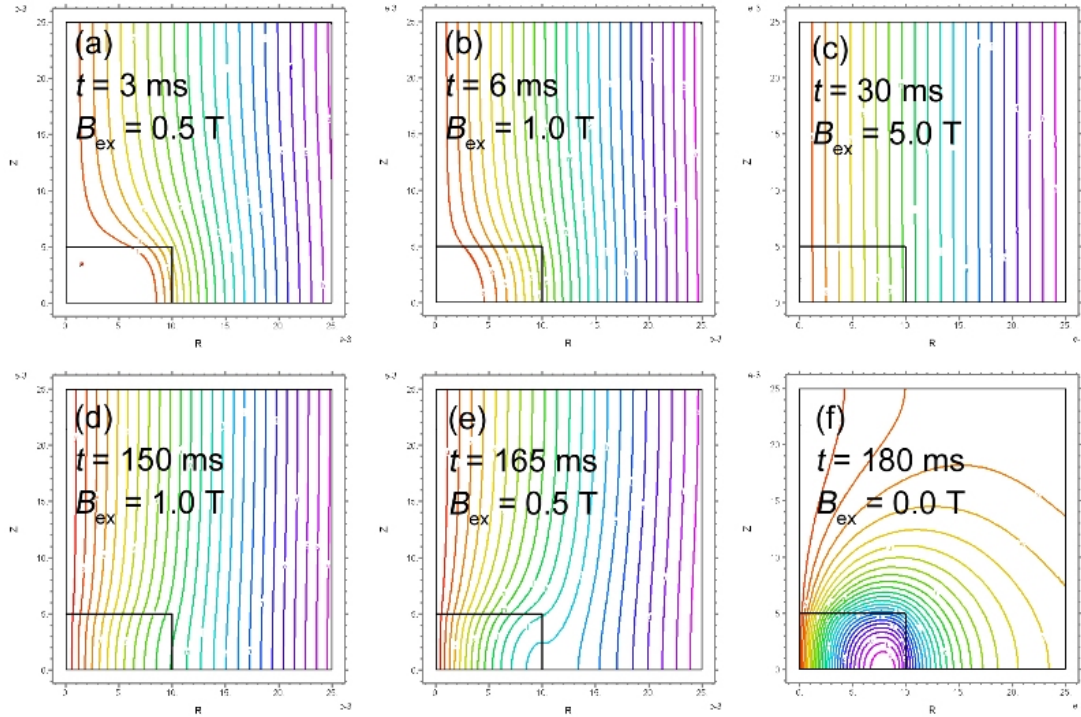


Figure 2. Simulation results of the distribution of magnetic field lines with the evolution of the PFM process. The large square in each part of the figure is 25 mm \times 25 mm and the small rectangle at the bottom-left corner is 10 mm \times 5 mm, representing a quarter of the cross section of the (RE)BCO bulk superconductor, i.e. each part is of the same dimension as that indicated in Fig. 1(b). The time and the corresponding magnitude of the pulse for each part of the figure are given.

Fig. 3 shows the contour maps of the current density $\mu_0 J$ normalised by the local $\mu_0 J_c$, so that the absolute value is either 1 (where the flux lines penetrate) or 0 (where there is no penetration), or by an intermediate value where a transitional state exists. Fig. 3(a) and (b) shows the increase of the penetration depth of the flux lines within the (RE)BCO bulk sample during the first 12 ms of the application of the pulse. The contour map of the current density at $t = 12$ ms implies particularly that the superconductor is fully penetrated under a field of ~ 2 T since the value is ~ 1 everywhere within the sample, which is consistent with the distribution of magnetic field lines at $t = 12$ ms. Fig. 3(c)-(f) indicates that the eddy current flowing in the superconductor reverses when the external pulse starts to descend from its peak value, as the dominant value switches from 1 to -1 . Hence the pulse in this example is considered large enough to penetrate the sample, and a remanent magnetic field will be trapped within the superconductor after the pulse is applied.

Fig. 4 shows the final distribution of the trapped field B_T and the current density $\mu_0 J$ along the radius near the top surface of the (RE)BCO bulk sample. It can be seen from Fig. 4(a) that the trapped field exhibits a well-defined conical distribution with a degree of curvature, which is in good agreement with the prediction of the Bean model [12]. The maximum value is 0.91 T at the centre of the top surface. Fig. 4(b) illustrates the distribution of current density within the sample, where Lines 1 (blue) and 2 (yellow) corresponds to the left and right hand sides of Campbell's equation, respectively. Line 2 superimposes perfectly onto Line 1, and both curves are consistent with the Kim model [11].

Fig. 4 implies that the assumptions of the present model are reasonable and that the solution converges at a sensible value.

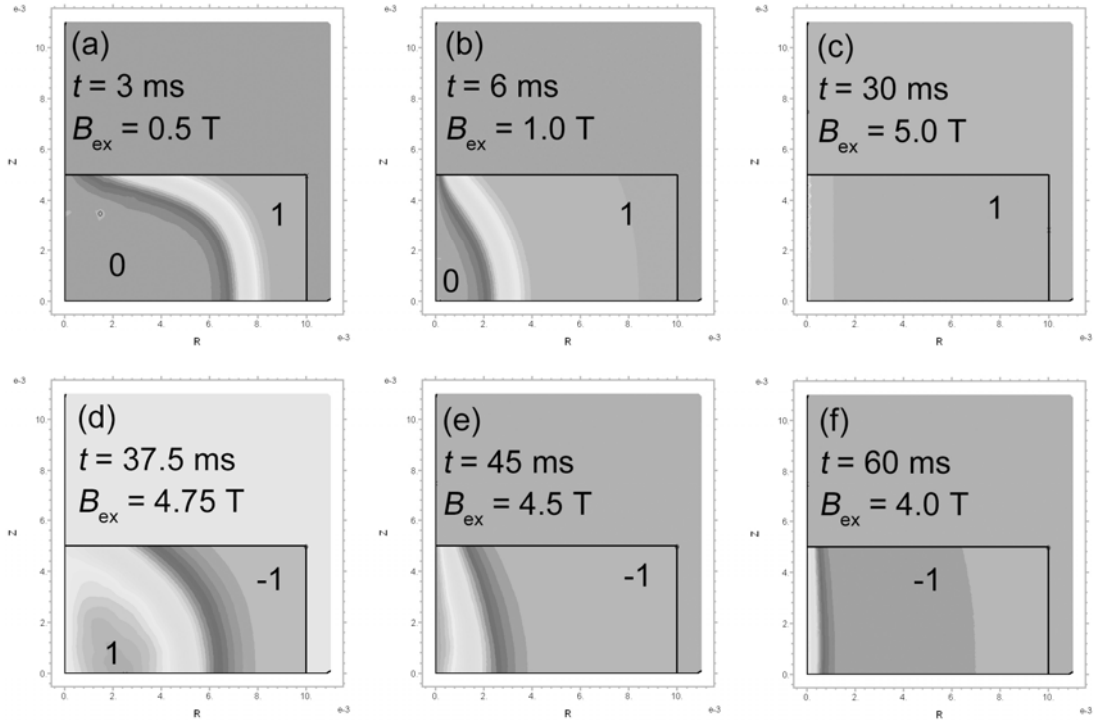


Figure 3. Simulation results of the distribution of current density $\mu_0 J$ within the (RE)BCO bulk sample with the evolution of the PFM process, normalised by the local $\mu_0 J_c$. Each part of the figure is amplified from the dimensions of Fig. 1(b) to $12.5 \text{ mm} \times 12.5 \text{ mm}$. The rectangle at the bottom-left corner still represents a quarter of the cross section of the (RE)BCO bulk superconductor.

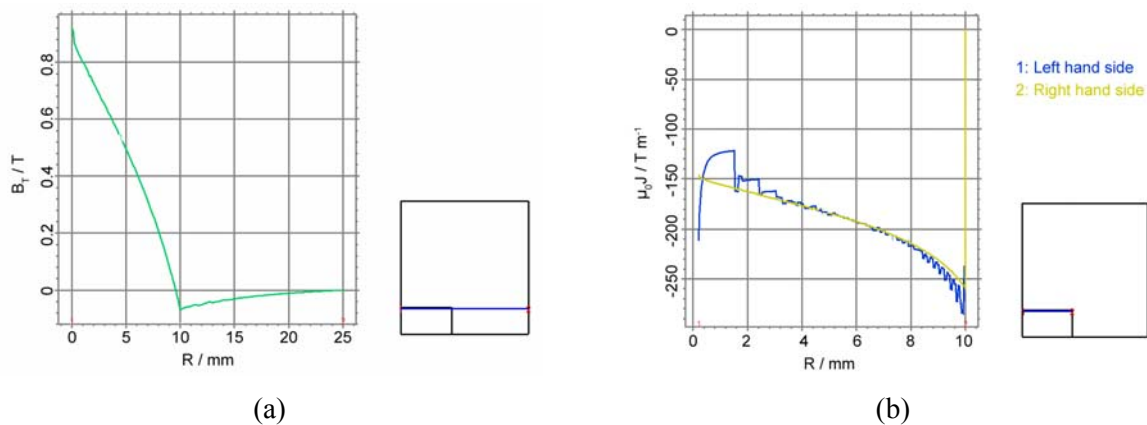


Figure 4. Simulation results of the final distribution of (a) the trapped field B_T and (b) the current density $\mu_0 J$ along the radius near the top surface of the (RE)BCO bulk sample after the pulse is applied.

The distributions of temperature change ΔT within the (RE)BCO bulk are illustrated in Fig. 5. It can be seen from parts (a) and (b) of this figure that the largest ΔT occurs at the upper-right corner of the sample, and that the temperature change is correlated strongly with the penetration of the magnetic

flux lines. Thereafter, a gradient of ΔT is established along the radial direction of the superconductor, which remains fairly steady until the end of the magnetisation process, as shown in Fig. 5(c)-(f). This gradient is associated probably with the gradient of magnetic field within the superconductor, so that the smaller field around the edge of the sample leads to a larger current density and a larger associated temperature rise.

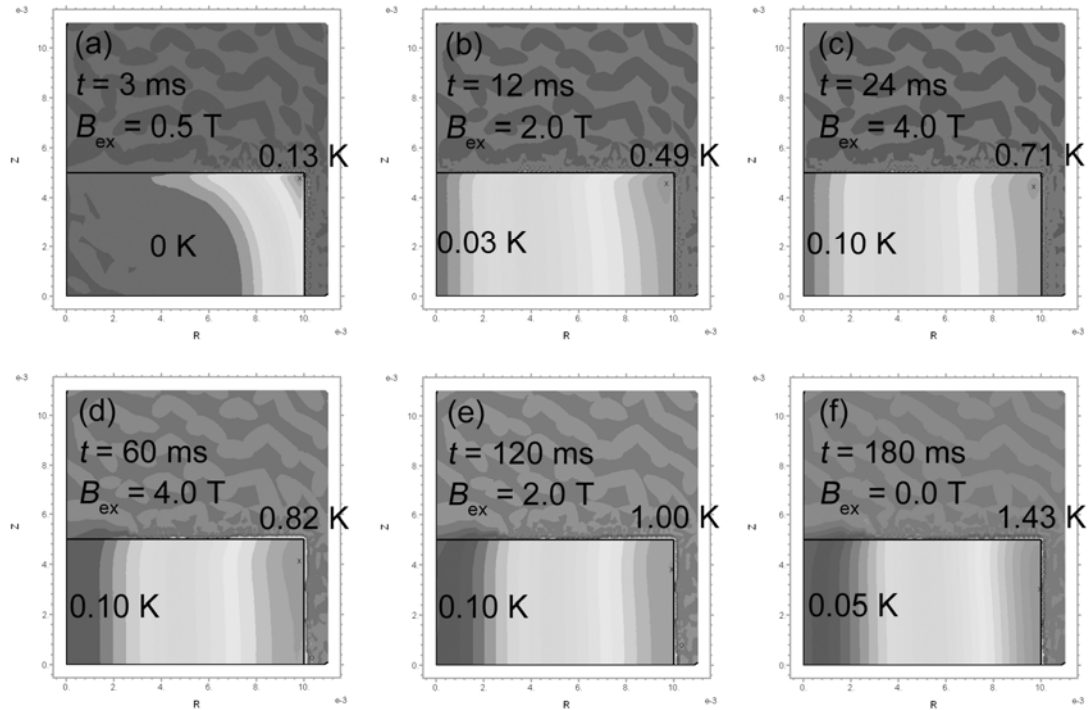


Figure 5. Simulation results of the distribution of temperature change ΔT within the (RE)BCO bulk superconductor with the evolution of the PFM process. Each part of the figure is of the same dimensions as those given in Fig. 3. The maximum and the minimum ΔT for each part are given.

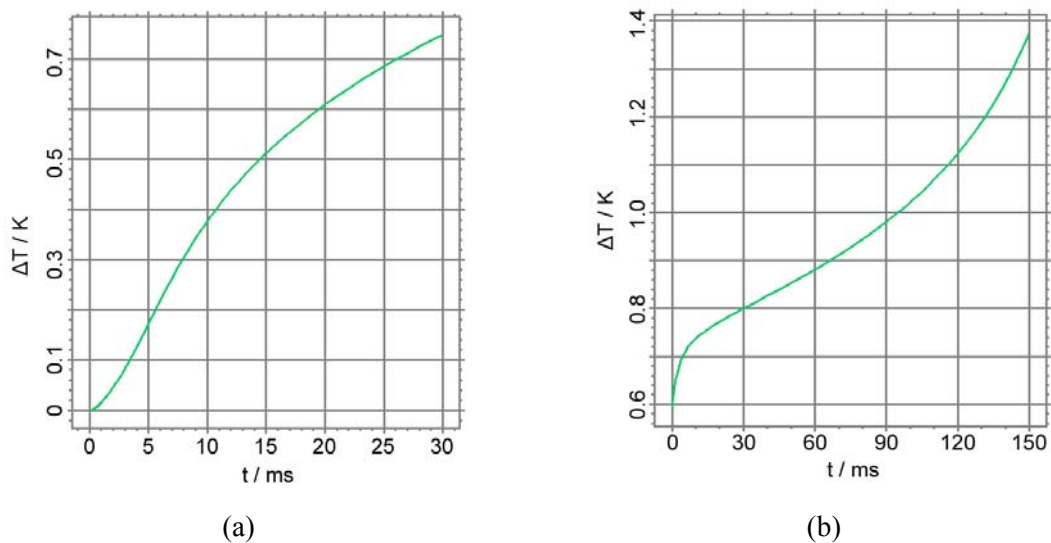


Figure 6. Simulation results of the time dependence of the temperature rise at the point (10,5) (in mm) during (a) the ascending stage and (b) the descending stage of the pulse.

The time dependence of the temperature rise at the point (10,5) (in mm) is summarised in Fig. 6, where parts (a) and (b) correspond to the ascending and the descending stages of the pulse, respectively. The temperature rise increases monotonically with the evolution of the process on the whole, apart from a slight discontinuity at $t = 30$ ms, corresponding to the split point of the pulse in this example. ΔT during the ascending and the descending stages is 0.75 K and 0.63 K, respectively, giving a total temperature rise of 1.38 K. The more rapid and larger temperature rise in the ascending stage is probably associated with the more rapid flux motion, since the ascending stage lasts for one-fifth of the time of the descending stage.

4. Conclusions

A framework of theoretical simulation has been developed to study the processes of pulsed field magnetisation of (RE)BCO bulk superconductors. Campbell's equation, the heat equation and the Kim model are included in the present model and solved simultaneously using the finite element method. The model calculates mainly the distribution of magnetic field lines, current density, trapped field, temperature change and heat flow within the superconductor. In the example given, the magnetisation of a (RE)BCO bulk (φ 20 mm / h 10 mm) by a single pulse (5 T / 30 ms / 150 ms) at 77 K is simulated, and the results of each stage of the process are recorded. Magnetic flux lines enter the superconductor mainly from the edge of the surfaces of the sample, and full penetration is achieved under an applied field of ~ 2 T. The temperature within the sample increases monotonically with the evolution of the process. After the pulse is applied, the trapped field exhibits a well-defined conical distribution with a degree of curvature, with the maximum value of 0.91 T at the centre of the top surface. Meanwhile a steady gradient of temperature rise is established along the radial direction of the superconductor, and the largest ΔT is about 1.4 K, occurring at the upper-right corner of the sample. Simulations for multi-pulse processes can be realised simply by running the model repeatedly. Hence the model may potentially serve as a cost-effective tool for the optimisation of the bulk geometry and the magnetisation profile in multi-pulse magnetisation processes.

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