Stable and Unstable Thermo – Current States of High Temperature Superconductors During Current Charging

Vladimir R. Romanovskii[1], Nikolay A. Lavrov, Vera K. Ozhogina

Russian Research Center 'Kurchatov Institute', Moscow 123182, Russia

E-mail: vromanovskii@netscape.net

Abstract – **We discuss formation peculiarities of the stable and unstable states in high-T_c superconductors. To understand the basic physical trends, which are characteristic for the current penetration mechanism in high-temperature superconductors, we investigate theoretically the operating states of Bi2212 slab without stabilizing matrix placed in DC external magnetic fields at low coolant temperature. We prove that the temperature of a high-T_c superconductor is not equal to the coolant temperature before instability onset. Therefore, the voltage-current characteristic of a high-T_c superconductor has only a positive slope during continuous current charging. As a result, it does not allow one to find the boundary between stable and unstable thermo – current states. This has to be taken into account during experiments where the critical current of high-T_c superconductors is defined.**

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I. INTRODUCTION

The boundary of current instability onset is important factor, which determines the limitations to the possible applications of superconductors. To estimate this quantity, the voltage-current characteristic (VCC) of a superconductor are widely used. For this purpose, the measurement of the critical current corresponding to the fixed electric field is made. This method is based on the Bean critical state model, which omits the non-linear part of the voltage-current characteristic. It is a satisfactory approximation for conventional low-T_c superconductors because their VCC is sufficiently steep. However, the voltage-current characteristics of high-T_c superconductors have a broad shape. As a result, high-T_c superconductors may operate at the currents that exceed their critical current [1-3]. Since the formulation of the current stability conditions should be based on the investigation of the stable and unstable operating modes of superconductor, in this study we discuss the formation peculiarities of the thermo-current states of high-T_c superconductor, which take place during current charging.

[1]Corresponding author

II. MODEL

Let us consider a superconductor with a slab geometry ($-a<x<a$, $-\infty<y<\infty$, $-b<z<b$, $b>>a$), which is placed in a constant external magnetic field parallel to its surface in the z-direction and is penetrated over its cross section ($S=4ab$). Suppose that the applied current is charged in the y-direction increasing linearly from zero with the constant sweep rate dI/dt and its self field is negligibly less than the external magnetic field. Let us describe the VCC of a superconductor by a power law and approximate the dependence of the critical current on the temperature by the linear correlation. Assume also that the superconductor has in the x-direction the transverse size, which does not lead to the magnetic instability. As it was shown in [4, 5], the flux penetration phenomena during creep are characterized by finite velocity. Therefore, taking into consideration the existence of the moving boundary of the current penetration region, the set of transient 1D equations describing the evolution of the temperature and electric field inside a superconductor is as follows:

$$C(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right)+\begin{cases}0, & 0<x<x_p\\ EJ, & x_p<x<a\end{cases} \tag{1}$$

$$\mu_0\frac{\partial J}{\partial t}=\frac{\partial^2 E}{\partial x^2},\quad t>0,\quad 0\le x_p<x<a \tag{2}$$

where the electric field $E(x,t)$, current density $J(x,t)$ and critical current density $J_c(T,B)$ conform the following relationships

$$E=E_c\left(\frac{J}{J_c(T,B)}\right)^n,\quad J_c(T,B)=J_{c0}(B)\frac{T_{cB}(B)-T}{T_{cB}(B)-T_0} \tag{3}$$

For the problem under consideration, the initial and boundary conditions are written as

$$\begin{gathered}T(x,0)=T_0,\quad E(x,0)=0,\quad \frac{\partial T}{\partial x}(0,t)=0,\ \lambda\frac{\partial T}{\partial x}(a,t)+h\left[T(a,t)-T_0\right]=0\\ \frac{\partial E}{\partial x}(a,t)=\frac{\mu_0}{4b}\frac{dI}{dt},\quad \begin{cases}E(x_p,t)=0, & x_p>0\\ \dfrac{\partial E}{\partial x}(0,t)=0, & x_p=0\end{cases}\end{gathered} \tag{4}$$

Here, C and λ are the specific heat capacity and thermal conductivity of a superconductor, respectively; h is the heat transfer coefficient; T_0 is the cooling bath temperature; n is the power law exponent of the E-J curve; E_c is the voltage criterion defining the critical current density of a superconductor; J_{c0} and T_{cB} are the known constant at the given external magnetic field B; x_p is the moving boundary of the current penetration region following from the integral relation

$$4b\int_{x_p}^{a}J(x,t)dx=\frac{dI}{dt}t \tag{5}$$

We used the finite difference method to solve the formulated problem. The simulation was made for a $Bi_2Sr_2CaCu_2O_8$ superconductor initially cooled at T_0=4.2 K, assuming that the heat removal conditions on the surface of the slab are close to the conduction-cooling condition.

The following constants B=10T, h=10^{-3}W/(cm^2×K), n=10, E_c=10^{-6}V/cm, T_{cB}=26.1K, J_{c0}=1.52×10^4A/cm^2 were set. The specific heat capacity and thermal conductivity of a superconductor were defined by the formulae

$$C(T) = 58.5\times10^{-5}T + 2.2\times10^{-5}T^3\left[\frac{J}{cm^3\cdot K}\right],\ T\le 10K$$
$$\lambda(T) = -1.234\times10^{-5} + 1.654\times10^{-4}T + 4.608\times10^{-6}T^2 - 1.127\times10^{-7}T^3 + 6.061\times10^{-10}T^4\left[\frac{W}{cm\cdot K}\right] \quad (6)$$

in accordance with [6, 7].

Besides, the evolution of the thermo-current states may be also estimated using static zero-dimensional model, if the primitive condition $dI/dt\to0$, $ha/\lambda<<1$ takes place. In the framework of the slab approximation considered, the corresponding current-voltage and temperature-voltage relations are described as follows on the next page [5]:

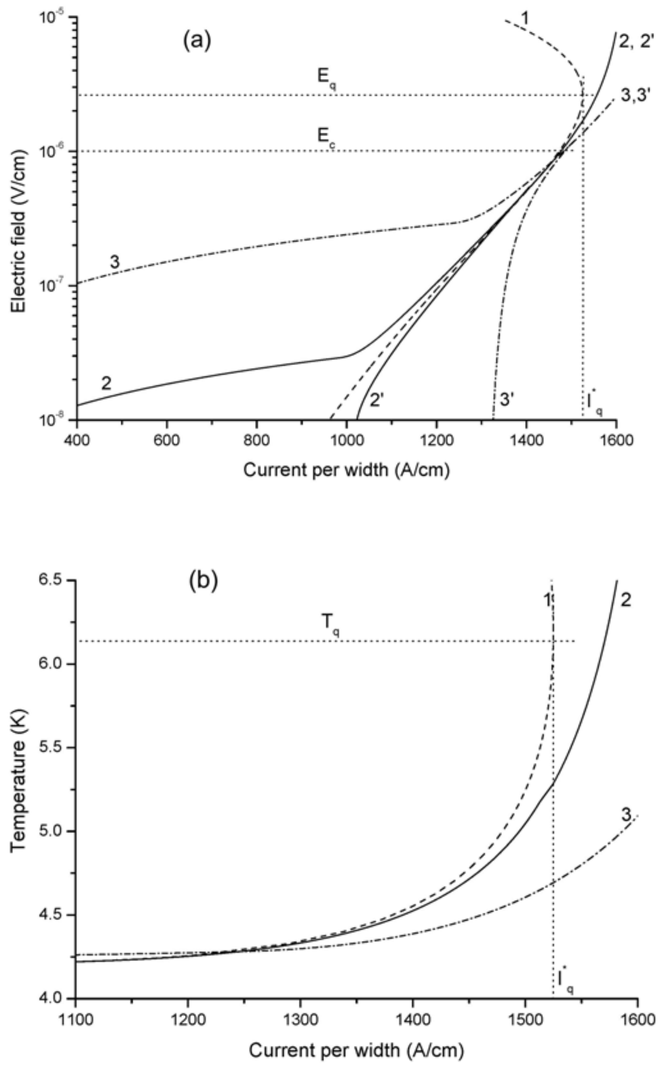

Fig. 1. Sweep rate dependence of the voltage-current (a) and temperature-current (b) characteristics of a high-T_c superconductor: 1 – $dI^*/dt\to0$; 2, 2´ – dI^*/dt=10^2A/(s·cm); 3, 3´ – dI^*/dt=10^3A/(s·cm). The values of E_q, T_q and I^*_q determine the boundary of the stable states.

$$J=\frac{J_{c0}}{1+\frac{J_{c0}}{E_c^{1/n}}\frac{a}{h(T_{cB}-T_0)}E^{\frac{n+1}{n}}}\left(\frac{E}{E_c}\right)^{1/n},\quad T=T_0+\frac{aJ_{c0}E}{h\left(1+\frac{J_{c0}}{E_c^{1/n}}\frac{a}{h(T_{cB}-T_0)}E^{\frac{n+1}{n}}\right)}\left(\frac{E}{E_c}\right)^{1/n} \qquad (7)$$

For convenience of the performed analysis, it was done using the current normalized by the slab width (I^*=0.5I^*/b) at a=5×10^{-2}cm. Thereby, the normalized current sweep rate was the single variable quantity: dI^*/dt=0.5$b^{-1}dI/dt$.

III. RESULTS

Figure 1 show the current dependence of the electric field and temperature induced in superconductor by current charging with two values of the normalized current sweep rate. Dashed curve 1 corresponds to a static uniform distribution of the electric field and temperature described by equations (7). Solid and dashed-dot curves (2, 2', 3, 3') are obtained using the model of Eq. (1) to (6). Here, the curves 2 and 3 correspond to the time variation of the thermo-current states of superconductor's surface and curves 2' and 3' describe the evolution of the thermo-current states of superconductor's centre.

The depicted curves indicate the existence of the following fully penetrated characteristic states that are proper for the stable and unstable current charging modes of high-T_c superconductors and depend on dI/dt.

First, there exists the transient period during which the fully penetrated electric fields on the surface of the slab and in its centre become practically equal. As a result, the redistribution time window between non-uniform and practically uniform fully penetrated states will decrease with increasing sweep rate.

Second, it is seen that in the over-critical electric field region (E>E_c) the fully penetrated dependences $E(I^*, dI^*/dt)$ and $T(I^*, dI^*/dt)$ that follow from the 1D unsteady model (1) – (6) are not identical with static dependences defined by equations (7). Moreover, the transient differential resistivity of the superconductor not only decreases with increasing dI^*/dt, but has just positive values. Consequently, the transient voltage-current characteristic of a high-T_c superconductor does not allow one to find the boundary of the current instability onset in the continuous current charging experiments. Really, in the static zero-dimensional approximation, the limiting current-carrying capacity is defined by the condition $\partial E/\partial J\to\infty$ [8], which identifies the boundary between positive (stable) and negative (unstable) static values of superconductor's differential resistivity. The corresponding static boundary quantities of the electric field E_q, current per width I^*_q and temperature T_q are depicted in Fig.1 by the dotted lines. According to the initial parameters used, these boundary points exist in the over-critical electric field region. The conditions describing the existence of the stable over-critical static states are formulated in [9]. However, the condition $\partial E/\partial J\to\infty$ is not observed in the unsteady thermo-current states as the transient voltage-current characteristic of a high-T_c superconductor has only positive values.

To explain this peculiarity, let us use the transient zero-dimensional approximation. In the fully penetrated mode, this approximation following from equation (1) is written as

$$C(T)\frac{dT}{dt}=-\frac{h}{a}(T-T_0)+EJ \qquad (8)$$

Assuming that the variation in the magnetic field does not significantly change the critical current of a superconductor, it is easy to get

$$\frac{dE}{dJ} = \frac{1 + \left|\frac{dJ_c}{dT}\right| \left(\frac{E}{E_c}\right)^{1/n} \frac{dT}{dJ}}{\frac{J_c(T)}{nE}\left(\frac{E}{E_c}\right)^{1/n}} \tag{9}$$

using the VCC of superconductor. To find the term *dT/dJ*, let us utilize equation (8) taking into consideration that

$$\frac{dT}{dt} = \frac{1}{S}\frac{dT}{dJ}\frac{dI}{dt}$$

Then unsteady dependence *dT/dJ* is given by

$$\frac{dT}{dJ} = \frac{EJ - (T - T_0)h/a}{C(T)\,{dI}/{dt}} S \tag{10}$$

Since *dT/dt*>0 during current charging then *dT/dJ*>0. Therefore, as follows from equations (9) and (10), the differential resistivity of a superconductor is always positive in the monotonously increasing current charging and the slope of the unsteady $E(I^*, dI^*/dt)$ and $T(I^*, dI^*/dt)$ dependences will become smaller when the heat capacity of a superconductor is higher, i.e., will decrease with increasing temperature of a superconductor during both stable and unstable states. As follows from Figure 1(b), there exists stable temperature rise of the superconductor before current instability. This unavoidable temperature variation of the superconductor will change its heat capacity and the discussed mechanism will be observed during current charging modes. Generally, in the case of superconducting composites, their allowable overheating is a function of the cooling conditions, amount of the superconductor in the composite, properties of matrix, etc. [10, 11].

To avoid the complexity of using both the static and unsteady definitions of the current instability boundary of high-T_c superconductors, the current charging with break is used in the experiments [12-18]. In this case the value *dI/dt* is equal to zero after the time when the applied current has a certain value I_0. Physically, this method is based on the existence of the static thermo-current states that precede the instability onset. For the first time it has been established in [19, 20] when investigating the ramp-rate limitation problem for low-T_c composite superconductor. They are described by the static voltage-current and temperature-current characteristics, which in simple cases satisfy equations (7). Therefore, all currents in Figure 1 which exceed the relevant value I^*_q correspond to unstable currents.

Consequently, in the monotonously increasing current charging modes, non-stationary voltage-current and temperature-current characteristics of high-T_c superconductors during fully penetrated states have only one branch with positive slope, which depends on the current sweep rate: the slope decreases when *dI/dt* increases. It exists both in the stable and unstable thermo-current states. This peculiarity is due to the temperature rise of a high-T_c superconductor both before and after instability onset that increases the heat capacity of the superconductor. That is why the transient voltage-current characteristics of high-T_c superconductors do not permit to find the current instability conditions and the possible stable increase in temperature of high-T_c superconductors should be taken into account for correct investigation of their critical currents.

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