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Fundamental Noise Processes in TES Devices

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***Abstract*—Microcalorimeters and bolometres are noise-limited devices, therefore, a proper understanding of all noise sources is essential to predict and interpret their performance. In this paper I review the fundamental noise processes contributing to Transition Edge Sensor (TES) microcalorimeters and bolometers and their effect on device performance. In particular, I will start with a simple, monolithic device model, moving to a more complex one involving discrete components, to finally move to today's more realistic, comprehensive model. In addition to the basic noise contribution (equilibrium Johnson noise and phonon noise), TES are significantly affected by extra noise, which is commonly referred to as "excess noise". Different fundamental processes have been proposed and investigated to explain the origin of this excess noise, in particular near equilibrium non-linear Johnson noise, flux-flow noise, and internal thermal fluctuation noise. Experimental evidence shows that all three processes are real and contribute, at different levels, to the TES noise, although different processes become important at different regimes. It is therefore time to archive the term "excess noise", considering them "fundamental noise processes" instead.**

***Index Terms*—Superconducting device noise, transition edge sensors.**

I. Introduction

The performance of microcalorimeters and bolometers is limited by noise [1],[2]. Noise sources can be divided in two main categories, fundamental and external. Fundamental sources are intrinsic to the device and cannot be removed, although the device can be optimized to reduce their effect. These include phonon noise, thermometer Johnson noise, and additional intrinsic thermometer noise. External sources are not directly attributable to the device and can be, at least in principle, reduced independently of the device parameters. These include the noise of the readout electronics and photon background noise.

Although they can be independently reduced, external noise sources are as important as fundamental ones to determine device performance. As an example, TESs were not widely used as device thermometers until high performance Superconducting Quantum Interference Devices (SQUIDs), with noise below the TES Johnson noise, became available in the 1990s.

In this paper I will focus on the fundamental noise in TES devices, and in particular on the intrinsic processes in TES thermometers. For a review of other noise terms, please refer to [1] and [2] and references therein.

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II. Monolithic Model: Phonon Noise and Equilibrium Johnson Noise

I will proceed with my review in steps which follow, roughly, a historical evolution of our understanding of TES devices. In this section I will start with a simple, monolithic model, affected by phonon noise between the device and the heat sink and equilibrium Johnson noise. In the next section I will move to a discrete component model, where internal phonon noise becomes important too, to finally focus on a more comprehensive model, where intrinsic thermometer noise is reevaluated.

The "modern" description of low temperature devices can be identified in a paper by John Mather who, in 1982 [3], reviewed the performance of non-equilibrium bolometers. The model was updated in 1984 to include microcalorimeters [4]. The model assumes that a device can be described as a single heat capacitance C at temperature T, linked to a heat sink at temperature T_S through a thermal conductance G (see Fig. 1). Joule power is dissipated in the readout thermometer, which is part of the capacitance C, raising its temperature above that of the heat sink. External power (bolometers) or energy (microcalorimeters) is absorbed by the capacitance, whose temperature changes are measured by the thermometer. The model assumes that these changes are small enough to use a linear approximation.

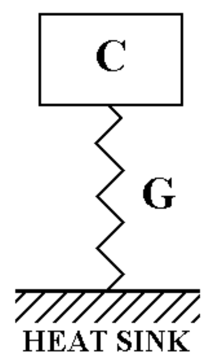

Fig. 1. A simple monolithic device can be schematically described as a heat capacitance C linked to a heat sink through a thermal conductance G. Joule power is dissipated in the capacitance, raising its temperature above that of the heat sink.

Two fundamental noise sources are identified in this model, thermal (power) fluctuations between the capacitance and the heat sink, called phonon noise (or thermal fluctuations noise), and Johnson noise of the readout thermometer.

The phonon noise can be described as a power noise on the detector and it has been quantified in a non-equilibrium system (assuming diffusive thermal conductivity) as [3]:

$$P_{ph} = \sqrt{4k_B G T^2}\left(\frac{\int_{T_S}^{T}\frac{\left(T'k(T')\right)^2}{\left(Tk(T)\right)^2}dT'}{\int_{T_S}^{T}\frac{k(T')}{k(T)}dT'}\right)^{\frac{1}{2}}, \quad (1)$$

where k_B is the Boltzmann constant, and *k(T')* the conductivity of the thermal link.

The Johnson noise in this model is being simply characterized as a voltage term across the thermometer resistance subject to the thermal response of the detector, namely the electro-thermal feedback [1]-[3], and the equilibrium expression for the voltage noise is:

$$e_J = \sqrt{4k_B TR}. \quad (2)$$

The figure of merit of calorimeters and bolometers is the noise equivalent power (NEP), defined as the power at the device input that generates an output equal to that generated by the noise. For bolometers, the NEP determines the minimum power that can be detected above the noise, while for microcalorimeters the NEP can be used to determine the detector energy resolution as [4]:

$$\Delta E_{FWHM} = 2.35\left(\int_0^\infty \frac{4df}{NEP^2}\right)^{-1/2}. \quad (3)$$

III. Discrete Component Model: Internal Phonon Noise

It is now accepted that the monolithic model, while quite powerful, does not fully describe the behavior of microcalorimeter and bolometers, especially at very low temperature, below ~300 mK. In the late 1990s, early 2000s, the model was updated to account for the fact that a real device is composed of several heat capacitances, connected with each other through internal thermal conductances [2],[5]-[7]. Different internal configurations have been proposed and investigated, for example, Fig. 2 shows an absorber which is thermally disconnected from the thermometer, and separate electron and phonon systems within each of them.

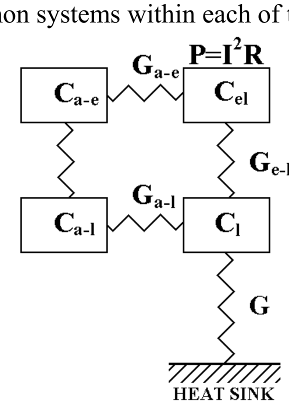

Fig. 2. A more realistic device is made of discrete components, linked with each other through internal thermal conductances. Here I show a model where the absorber and thermometer are thermally separate, each of them has a separate electron and phonon system, and the link to the heat sink is done through the thermometer. Other geometries are possible (see for example [7]).

To fully characterize a discrete component device, an additional phonon noise term must be included for each internal conductance. The model becomes immediately more complex, comprising of a separate differential equation for each discrete component. The system of differential equations can be solved numerically [6] or algebraically using a linear approximation [2],[7].

IV. Comprehensive Model: Intrinsic TES Thermometer Noise

While the discrete model seems to explain quite well the behavior of thermistor-based devices [2], as soon as research groups started modeling the behavior of TES devices it was clear the presence of additional noise, dubbed "excess noise" [8]-[10]. The term comes, in part, from the fact that it was originally believed that such noise contribution was not intrinsic to TES detectors and could be eliminated. It is now clear that, in fact, the additional noise comes from fundamental processes in TES devices. While a good knowledge of its behavior is necessary for the optimization of devices to minimize its effect, it cannot be completely removed. Moreover, so far I used the singular to describe this additional noise, but it is, indeed, due to the contribution of different effects. In particular, processes that may contribute to it are near equilibrium non-linear Johnson noise, flux-flow noise, internal thermal fluctuation noise, and percolation noise [5],[11]-[13]. I will focus here on the first three terms, which have been identified in TES data.

A. Near Equilibrium non-linear Johnson Noise

It turns out that perhaps the biggest additional contribution to the noise at the working point of many devices is not due to a separate noise term, but to the incomplete description of Johnson noise. While the theory for phonon noise in a non-equilibrium system was developed in 1982 [3], the equilibrium model for Johnson noise was used, basically unchallenged, for another 2 decades. The reason lies, in part, in the difficulty of the calculations and the fact that the equilibrium theory describes quite well the behavior or thermistor-based devices [2]. It was proposed at the first Workshop on TES device physics in 2002 that the equilibrium Johnson noise used to model the device performance was not necessarily correct and that a more realistic theory for Johnson noise in a non-equilibrium system was lacking.

The first thermodynamically correct calculation of the noise in a simple nonlinear resistive bolometer or calorimeter operated out of equilibrium was performed by Kent Irwin in 2006 [11]. The solution is rigorous for first and second order deviations from equilibrium, and for the linear and quadratic terms of dissipative elements. Using only the first order terms of the solution, the Johnson voltage noise can be written as:

$$e_J = \sqrt{4k_B TR(1+2\beta)}, \quad (4)$$

where $\beta = (d\log R \,/\, d\log i) = i/R\; dR/di$.

A common practice to quantify intrinsic noise contributions is the use of the parameter M, defined as the ratio between an intrinsic noise term and the high frequency limit of (equilibrium) Johnson noise ($\sqrt{4k_B TR}$). The near equilibrium non-linear contribution to Johnson noise is represented by the factor 2β in the square root, and the parameter M is simply:

$$M_\beta = \sqrt{2\beta}. \tag{5}$$

I should point out that, since the non-equilibrium theory has been verified and commonly accepted, the parameter M has been redefined as the ratio between an intrinsic noise term and the high frequency limit of Johnson noise $\left(\sqrt{4kTR(1+2\beta)}\right)$, however, for direct comparison between noise processes, I will use the "older" definition of M in this paper.

It is also possible to write M_β in terms of TES working parameters in the framework of Ginzburg-Landau Theory [14]. In particular, by using the critical current-critical temperature relation for a superconductor

$$T = T_o\left(1 - \left(\frac{I}{I_o}\right)^{\frac{2}{3}}\right), \tag{6}$$

with T_o and I_o are the zero current critical temperature and the zero temperature critical current respectively. Using Eq. 6 the parameter β can be written in terms of the detector sensitivity $\alpha = d\log R \,/\, d\log T = T/R \;\; dR/dT$ as:

$$\beta = \frac{2}{3}\,\alpha\,\left(\frac{I}{I_o}\right)^{\frac{2}{3}}, \tag{7}$$

where I is the device bias current. The parameter M_β then becomes:

$$M_\beta = \frac{2}{\sqrt{3}}\,\sqrt{\alpha}\left(\frac{I}{I_o}\right)^{\frac{1}{3}}, \tag{8}$$

Notice that, while the sensitivity α also loosely depends on the bias current [15], at constant α the parameter M_β depends only on the bias current I. In particular, it does not depend on the device resistance R.

B. Flux Flow Noise

Flux flow noise goes well beyond TES devices and has been studied in details in superconducting films near the transition. The idea is relatively simple, vortexes in a superconductor near the transition move perpendicular to current flow with speed:

$$v = \frac{\phi_o J}{\eta}, \tag{9}$$

where ϕ_o is the quantum flux, J the current density, and the viscosity η is given by

$$\eta = \frac{\phi_o \mu_o H_{C2}}{\rho_n}. \tag{10}$$

The parameters μ_o and H_{C2} are the magnetization of vacuum and the critical field respectively. In its motion, each vortex generates a voltage

$$V_1 = \frac{v\phi_o}{w} = \frac{\phi_o I R_n}{\mu_o w d H_{C2}}, \tag{11}$$

where R_n is the normal resistance of the film, w its width, and d its length. If the resistance at the transition is solely due to flux flow motion it can be written as $R = V/I = NV_1/I$, and the number of vortexes crossing the film is:

$$N = \frac{RI}{V_1} = \frac{\mu_o}{\phi_o}\frac{R}{R_n} w^2 H_{C2}. \tag{12}$$

For small N this generates a voltage noise $V_{RMS} = \sqrt{N} V_1 = \sqrt{RIV_1}$, and a noise power spectrum $S_{ff} = V_{RMS}^2/B$, where $B = v/l$ is the vortexes bandwidth, with l the vortexes mean free path. It has also been shown empirically that the noise "saturates" above a certain voltage V_o, possibly due to correlation between vortexes [16] or other effects contributing to the device resistance. An empirical expression for the voltage contribution for flux flow noise therefore is:

$$e_{ff} = \sqrt{\phi_o \frac{V}{1+V/V_o}\frac{l}{w}}. \tag{13}$$

With the simple assumption that the vortexes mean free path is inversely proportional to the number of vortexes, it can be shown that $l/w \propto 1/R$, and the noise factor M can be written as:

$$M_{ff} = \frac{1}{\sqrt{4k_BTR}}\sqrt{\phi_o \frac{V}{1+V/V_o}\frac{R_{ff}}{R}} = \frac{1}{\sqrt{4k_BTR}}\sqrt{\phi_o \frac{IR_{ff}}{1+IR/V_o}}, \tag{14}$$

where R_{ff} is a constant proportionality coefficient with units of Ohms that we introduced to simplify out expression ($l/w = R_{ff}/R$). Using data from NASA/GSFC [17], we get values of $V_o \sim 3\times10^{-7} V$ and $R_{ff} \sim 1\ \mu\Omega$. We point out that the value of R_{ff} is inversely proportional to the critical field H_{C2}.

C. Internal Thermal Fluctuations Noise

In section III I briefly introduced the internal phonon noise due to a discrete component device. In fact, the idea can be pushed even further to include thermal fluctuations within the TES. A TES, in fact, can be considered as a thin film with its resistance distributed through an area $w \times d$. If the film is uniform, Joule power is dissipated evenly throughout the film, but even in the best case scenario there will be power, and therefore temperature fluctuations between different regions of the film. Phonon noise may thus be generated inside the film itself, called Internal Thermal Fluctuations Noise (ITFN).

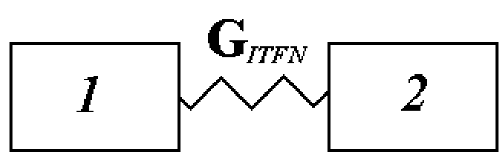

Fig. 2. The simplest model for internal thermal fluctuations assumes two systems, 1 & 2 connected by a thermal conductance G_{ITFN}.

The simplest possible model to describe this effect is by using two elements connected by a conductance G_{ITFN} (see Fig. 3). While the ITFN is really due to a distributed system of elements and conductances, its effect can be described by this simple model with the proper choice for G_{ITFN} [18],[19]. A complete calculation for the ITFN for a two element system can be found in [5]. A simple expression can be derived if the two systems 1 & 2 are at the same temperature. Then the thermal power fluctuations is simply the equilibrium expression $\Delta P = \sqrt{4k_B G_{ITFN} T^2}$, which generates a temperature fluctuation at each end $\Delta T = \Delta P / G_{ITFN}$. Using the definition of detector sensitivity α this can be converted into a voltage fluctuation:

$$\Delta V = I\Delta R = \alpha RI \sqrt{\frac{4k_B}{G_{ITFN}}}, \quad (15)$$

which corresponds to:

$$M_{ITFN} = \alpha \sqrt{\frac{P}{G_{ITFN}\, T}}. \quad (16)$$

This is the same expression derived in [19] with a more formal treatment. We can also explicitly write out the power in terms of current and resistance and use Wiedemann-Franz law to write the internal conductance of the TES in terms of its resistance $G_{ITFN} = LT/R$, where L is the Lorenz number. The parameter M_{ITFN} then becomes:

$$M_{ITFN} \approx \alpha \sqrt{\frac{I^2 R^2}{AL\, T^2}} = \frac{\alpha IR}{T} \frac{1}{\sqrt{AL}}, \quad (17)$$

where the parameter A is just a normalization term to account for the fact that conductance responsible for the ITFN is bigger, but proportional, to the total conductance from one end to the other of the TES, or, in other words, it is used to scale the simple one-conductance approximation to a more realistic description of the TES. Using data from the Netherland Institute for Space Research (SRON) [18],[19], the value of A seems to be of the order of 8.

Notice that, for constant α, the effect of the ITFN is proportional to the voltage bias of the device $V = IR$.

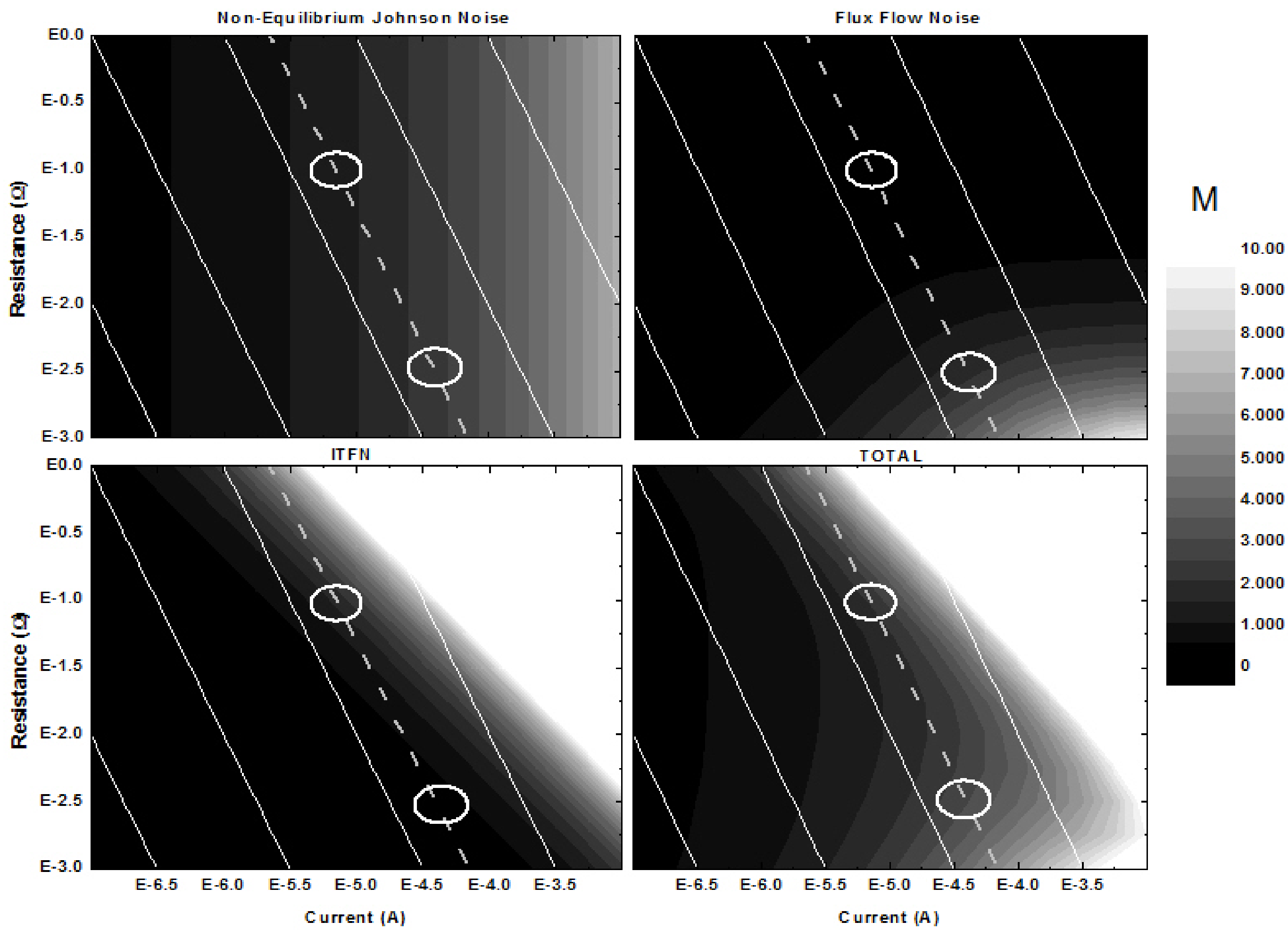

Fig. 4. Noise factor M for different effects and their sum, as a function of current and resistance. The white lines represent constant power curves, the dashed line correspond to the constant power of 5 pW. The ellipses represent typical working points for high resistance and low resistance devices.

D. Comparison of different effects

Using the relatively simple expressions described in the previous sections (Eqs. 8, 14, and 17) it is possible to compare the effect of the different fundamental noise terms and optimize a TES device. As an example, Fig. 4 shows the noise parameter M as a function of current and resistance at the working point for a constant value of the sensitivity α of 100. The device parameters used to generate the data are a temperature T=0.1 K, zero temperature critical current

$T_o = 5mA$, $R_{ff} = 1\mu\Omega$, $V_o \sim 3 \times 10^{-7} V$, and $A = 8$. The white lines in the plots correspond to constant power curves, the dashed lines correspond to the specific constant power of 5pW, typical of X-ray TES microcalorimeters, and the two white regions are typical working points for TES microcalorimeters, corresponding to low-R and high-R devices.

While I have already discussed the overall characteristics of each noise process, let me point out a few general trends from the plots. First of all, as pointed out before, different noise processes become important at different regimes. However, flux flow noise seems rather small and negligible at most TES working regimes, although its effect may become important if working at very low resistance and high current, such as at the very beginning of the TES transition. It may become also important for very low α devices, as the other terms will decrease while flux-flow stays the same.

The effect of non-equilibrium Johnson noise and ITFN is rather complementary. Using as an example the 5pW dashed curves, high resistance devices have lower non-equilibrium Johnson effect and higher ITFN, while the proportion is reversed for lower resistance devices. The net effect, however, is similar for both working regimes. Remember, however, these plots are made at constant sensitivity, and the three processes have different dependence on α.

E. Additional Noise Contributions

Before concluding my review, I would like to point out that this is not, by far, the last word on TES fundamental noise processes. While these three contributions have been identified in TES devices, and likely represent most of what used to be called excess noise, additional terms may be necessary to describe the device performance once these three effects are properly characterized. In particular, we should expect to have higher terms in the near-equilibrium non-linear Johnson noise [11] that could become important at specific working regimes, and statistical models [13], [20] are currently being investigated.

V. Conclusion

Transition edge sensors are affected by different fundamental noise effects. Some of them are well studied and characterized, while others have been identified but still require a significant amount of characterization. It seems quite clear at this point, however, that what in the past decade has been dubbed "excess noise" is now mostly attributable to fundamental processes in the TES devices, so perhaps it is time to archive the term.

Acknowledgment

The author would like to thank the many researchers that have worked on TES noise during the course of the last ten years. The list of references in this paper is far from exhaustive, so a special thank to those that have not been specifically mentioned.

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