IEEE/CSC & ESAS European Superconductivity News Forum (ESNF), No. 15, January 2011
The published version of this manuscript appeared in *IEEE Transactions on Applied Superconductivity* **21**, Issue 3, 176 - 179 (2011)

Shapiro Step Response of Intrinsic Josephson Junctions with High Critical Currents of $(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$

Nurmiza Binti Othman, Gin-ichiro Oya, Michihide Kitamura, and Akinobu Irie

***Abstract*—We have studied the response of intrinsic Josephson junctions (IJJs) in cross-shaped samples of $(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$ (x = 0.15) with high critical currents I_c at 4.2 K to injection of microwave with frequencies f_{rf} of 2–20 GHz. At the early stage of measurements Josephson vortex flow is induced in the IJJs by supplying high currents to them. After that, by injection of microwave power *P* to them clear constant voltage steps are successfully observed on their current-voltage characteristics, although their plasma frequency f_{pl} is much higher than f_{rf} and they are not resistively shunted. The constant voltage steps appear so as to satisfy the Josephson frequency-voltage relation and behave like Shapiro steps depending on *P*. Such behavior of steps is well reproduced by numerical simulations on Shapiro step response of JJs with shunt resistivity which is equal to the Josephson-vortex flow resistivity under microwave injection. Consequently, the observed constant voltage steps may be Shapiro steps out of the IJJs with the Josephson-vortex flow resistivity.**

***Index Terms*—$(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$, high-$T_c$ superconductor, intrinsic Josephson effect, Shapiro step.**

I. Introduction

SINCE the discovery of the intrinsic Josephson effect in high-T_c cuprate superconductors such as $Bi_2Sr_2CaCu_2O_y$ (BSCCO) and $(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$ (BPSCCO), numerous studies of the basic nature, nonlinear dynamics and device applications of the superconductors have been extensively developed [1], [2]. Recently, emissions of coherent THz radiation from the intrinsic Josephson junctions (IJJs) of BSCCO in addition to detection of THz radiation by them have attracted much interest [3]–[5]. Nevertheless, practical devices are expected to cover a relatively wide frequency range of GHz–THz in a wide temperature range of 4.2 K–~T_c.

Recently, the conditions for an observation of Shapiro steps from an IJJ and its stack of BSCCO have been numerically studied considering the *d*-wave symmetry superconducting gap, thermal noise and low external magnetic field by Kitamura *et al*. [6], [7]. And, it has been confirmed that Shapiro steps can be observed stably and clearly from the IJJ and its stack under the conditions similar to those for conventional low-T_c Josephson junctions. Therefore, in order to observe non-chaotic behavior of Shapiro steps out of the Josephson junction with plasma frequency f_{pl} under irradiation of microwave of frequency f_{rf}, it may be necessary to fulfill the general conditions, either $f_{rf} > f_{pl}$ or $f_{rf} < f_{RC}$ (where $f_{pl} = (eI_c/h\pi C)^{1/2}$ and $f_{RC} = 1/2\pi RC$, in which *e*, *h*, I_c, *C* and *R* are the electronic charge, Planck's constant, the critical current of the junction, the junction capacitance, and the junction resistivity, respectively), or the junction is resistively shunted, according to Kautz *et al*. [8]. Actually, some research groups have successfully observed Shapiro steps from the inner IJJs, the surface IJJs or resistively shunted IJJs of BSCCO under the above mentioned conditions [4], [5], [9]–[11].

On the other hand, the IJJs of BPSCCO may be better suited for operation in a THz range than those of BSCCO, because the former has much higher critical current densities J_c than the latter up to ~T_c [12]. However, it may be also desired for them to operate over a wider frequency range of GHz–THz. From this point of view, in the present work we have studied the response of the IJJs of BPSCCO to microwave injection at f_{rf} between 2 and 20.5 GHz much lower than their f_{pl} at 4.2 K. Then, we have successfully observed clear constant voltage steps on the current-voltage (*I*–*V*) characteristics of the IJJs with high I_c in which the resistive Josephson-vortex flow is induced by the current supplied to the IJJs.

In this paper we show the experimental results on the microwave responses of the IJJs of BPSCCO, numerically calculated results corresponding to them, and then compare and discuss the two results, and finally conclude the observed steps to be Shapiro steps.

II. Theoretical Background

Recently, Machida and Sakai have developed a unified theory containing both electric and magnetic field couplings between neighbors in multistacked Josephson junctions [13]. Then, Kitamura *et al*. have further evolved this theory and numerically studied the condition for the observation of Shapiro steps in BSCCO IJJs, considering the *d*-wave symmetry superconducting gap and thermal noise also [7].

According to them, for a stacked system consisting of *N* identical superconductor-insulator-superconductor Josephson junctions along the *z*-axis direction, with an external magnetic

Manuscript received 3 August 2010. This work was supported in part by the Grant-in-Aid for Science Research from the Ministry of Education, Science, Sports and Culture of Japan, and by Universiti Tun Hussein Onn, Malaysia.

N. B. Othman was with Utsunomiya University, Utsunomiya 321-8585, Japan. She is now with the Department of Electrical and Electronic Engineering, Universiti Tun Hussein Onn, Parit Raja 86400 Johor, Malaysia.

G. Oya, M. Kitamura and A. Irie are with the Department of Electrical and Electronic Systems Engineering, Utsunomiya University, 7-1-2 Yoto, Utsunomiya 321-8585, Japan (e-mail: g_ioya@cc.utsunomiya-u.ac.jp).

field parallel to the y-axis direction, the gauge-invariant phase difference $\varphi_l(x,t)$ of the lth junction, which is assumed to be a function of spatial variable x and real time t, satisfies the following coupled sine-Gordon equation with a matrix form:

$$\Sigma_C \lambda_J^2 \frac{\partial^2}{\partial x^2} \begin{pmatrix} \varphi_1(x,t) \\ \vdots \\ \varphi_l(x,t) \\ \vdots \\ \varphi_N(x,t) \end{pmatrix} = \Sigma_L \begin{pmatrix} J_1(x,t) \\ \vdots \\ J_l(x,t) \\ \vdots \\ J_N(x,t) \end{pmatrix}, \qquad (1)$$

where λ_J is the Josephson penetration depth given by $(\Phi_0/2\pi\mu_0 d'_L J_c)^{1/2}$ using a flux quantum Φ_0, the vacuum permeability μ_0, and the critical current density J_c. The Σ_C and Σ_L are matrices describing the electric and magnetic interactions between neighboring Josephson junctions and are written as

$$\Sigma_{C(L)} = \begin{pmatrix} 1 & \Sigma_{C(L)} & 0 & \cdots & \cdots \\ \Sigma_{C(L)} & 1 & \Sigma_{C(L)} & 0 & \cdots \\ \cdots & \Sigma_{C(L)} & 1 & \Sigma_{C(L)} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & 0 & \Sigma_{C(L)} & 1 \end{pmatrix}, \qquad (2)$$

where the coupling constant Σ_C and Σ_L are given by

$$\Sigma_{C(L)} = -\frac{\lambda_{e(L)}}{d'_{C(L)} \sinh\left(w/\lambda_{e(L)}\right)}. \qquad (3)$$

The d'_C and d'_L are the effective electric and magnetic thickness given by

$$d'_{C(L)} = d + 2\lambda_{e(L)} \coth\left(w/\lambda_{e(L)}\right), \qquad (4)$$

using electrode thickness w and barrier thickness d. The λ_e and λ_L are the Debye screening length and the London penetration depth, respectively. The current $J_l(x,t)$ in Eq. (1) can be written in SI units as

$$J_l(x,t) = \left(\frac{\Phi_0}{2\pi}\right)\frac{C_u^*}{J_c}\frac{\partial^2\varphi_l(x,t)}{\partial t^2} + \left(\frac{\Phi_0}{2\pi}\right)\frac{G_u^*}{J_c}\frac{\partial\varphi_l(x,t)}{\partial t} + \sum_{l'=1}^{N}\{\Sigma_c\}_{l,l'} i_{CP}^{(l')}(x,t) - i_{ext}(t), \qquad (5)$$

where C^*_u and G^*_u are the effective unit area capacitance $\varepsilon\varepsilon_0/d'_c$ and conductivity σ/d'_c per a junction, respectively. The current $i^{(l)}_{CP}(x,t)$ is the normalized Cooper Pair tunneling current of the lth Josephson junction. The normalized external current $i_{ext}(t)$ is given by

$$i_{ext}(t) = i_0 + i_r \sin\omega_r t + i_{noise}(t), \qquad (6)$$

where i_0 is the normalized external dc current, $i_r \sin\omega_r t$ is the normalized external ac modulation current with the frequency ω_r, and $i_{noise}(t)$ is the normalized current due to thermal noise.

III. Experimental

The BPSCCO single crystals with Pb content of x = 0.15 were grown by using a conventional melting method reported elsewhere [14]. Platelets of the single-crystals were glued onto glass substrates and cleaved in air, and then Au films of ~50 nm thickness were deposited on the cleavage surfaces. After that, cross-shaped samples with lateral dimensions of (4.5–6.0) μm × (6.5–8.5) μm were fabricated in the platelets by applying standard photolithography and ion etching successively to either surface of them [11]. Additionally, a gold stripline was formed on the top of each sample to inject microwave.

The samples with T_c of 80–85 K were cooled down to 4.2 K by liquid helium. Then, their *I*–*V* characteristics along the *c*-axis direction were measured by supplying currents much higher than their critical currents I_c. And then, microwaves of f_{rf} between 2 and 20.5 GHz were injected from a microwave source to the samples through the striplines, so that the responses of IJJs were examined.

IV. Results and Discussion

Fig. 1 shows the *I*–*V* characteristic of a stack of 17 IJJs of a sample with the junction area S of ~50 μm^2 at 4.2 K. This is characterized by high I_c of ~5 mA and a large hysteresis but strongly reduced by self-heating of the sample. The inset shows the enlarged upper part of its 0th branch. We can see a resistive branch on the top of it. This may be caused by the Josephson-vortex flow in the sample. In this case, Josephson vortices may be induced and motivated to flow in some of the IJJs in it by the applied current I much higher than I_c without external magnetic field. Such resistive branches are also observed from other samples. Then, the behavior of the 0th branch accompanied with the resistive branch is studied with injection of microwave power into each sample.

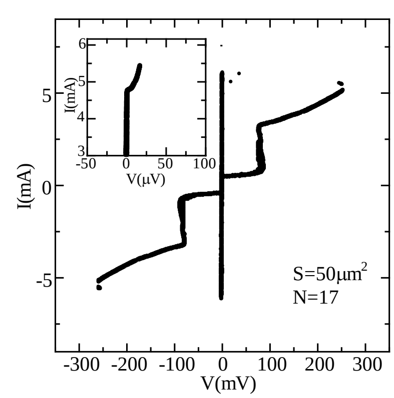

Fig. 1. *I*–*V* characteristic of a sample at 4.2 K. The inset shows the vortex-flow

branch on the top of the 0^{th} branch.

In this study we have successfully observed two pronounced behavior of the 0th branch of the *I–V* characteristic of the injected sample. One is a change of inclination of the resistive branch, that is, Josephson-vortex flow resistivity R_{fl}. Another is an appearance of constant voltage steps on it, with a decrease in I_c. Fig. 2 shows the typical behavior of the constant voltage steps on the 0th branch of sample 1 (with S of ~36 μm^2) under injection of microwave power P of 1.0, 1.6 and 4.0 mW at f_{rf} of 10 GHz. Here, we roughly estimated R_{fl} to be ~75 mΩ with a low power of microwave, but ~43 mΩ without microwave, from the corresponding *I–V* characteristics. Now, remarkably, constant voltage steps appear at intervals of ΔV ~20.5 μV, up to the fourth order. The height of these steps remains nearly constant as P increases to 4.0 mW. The voltage V_n of the nth step nearly satisfies the Josephson frequency-voltage relation $V_n = nhf_{rf}/2e$ for a JJ, where n = 0, 1, 2, 3 and 4. Additionally, under injection of microwaves of 5 and 15 GHz similar behavior of the 0th branch is also observed so that the constant voltage steps appear at ΔV of ~10.3 and ~31.0 μV to satisfy the above Josephson frequency-voltage relation for a JJ. To confirm such behavior of the 0th branch, similar studies are further done for other samples under injection of microwaves at some f_{rf}. As a result, the constant voltage steps appear also on the 0th branch of any sample, satisfying the Josephson frequency-voltage relation. Thus, the observed steps behave like Shapiro steps out of an IJJ in the sample. However, in these cases the standard conditions to observe Shapiro steps don't seem to be satisfied, because f_{pl} of the sample is much higher than f_{rf} and also no shunt resistor is externally attached to the sample.

In addition, we succeed in inducing Josephson vortices in two IJJs in sample 3 (with S of ~50 μm^2) and doubling its R_{fl} by controlling the current to the sample. Fig. 3 shows the 0th branches of the *I–V* characteristics of the sample under injection of microwave at f_{rf} of 7 GHz as a function of P. In this case, constant voltage steps with some rounding appear at ~29 μV so as to satisfy roughly the relation $V_{mn} = mnhf_{rf}/2e$ considering the number of phase-locked junctions m, where m = 2 and n = 0 and 1. This means that the two IJJs behave so as to synchronize with each other under injection of microwave.

To define the nature of the observed constant voltage steps, their ΔV related to m and f_{rf}, together with I_c and R_{fl} (without microwave injection) of the samples are summarized in Table I.

TABLE I SAMPLE PARAMETERS

Sam-ple No.	I_c (mA)	R_{fl} (mΩ)	m	f_{rf} (GHz)	ΔV (μV)
1	4.5	43	1	5	10.3
				10	20.5
				15	31.0
2	6.8	34	1	10	20.5
				15	31.0
3	5.3	42	1	5	10.3
		83	2	7	29.0

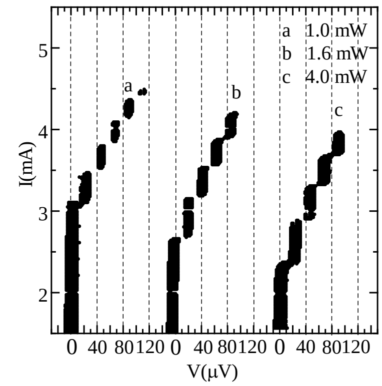

Fig. 2. The behavior of the 0^{th} branch of the *I–V* characteristic of sample 1 with injection power of microwave of f_{rf} of 10 GHz.

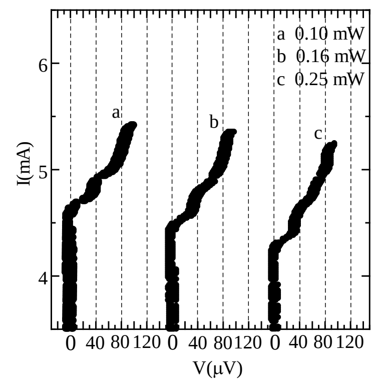

Fig. 3. The behavior of the 0^{th} branch of the *I–V* characteristic of sample 3 with injection power of microwave of f_{rf} of 7 GHz.

We now notice that both ΔV and R_{fl} depend on m. This implies that one IJJ or two accompanied with Josephson-vortex flow in the samples respond to the injected microwave. Therefore, R_{fl} seems to play an important role for the IJJ to respond to the microwave, or to satisfy the condition to observe Shapiro steps from it.

From this point of view, we assume that R_{fl} acts as the shunt resistivity R_{shunt} under the injection of microwave, because they seem to behave in the same manner on the *I–V* characteristics. On this assumption, we study numerically the microwave response of resistively shunted JJs corresponding to the above mentioned IJJs by using (1)–(6), and compare the experimental results with the numerical results to make the behavior of the IJJs under microwave injection clear.

First, to know the basic behavior of the IJJ under microwave injection, we calculate the *I–V* characteristics of the JJ corresponding to that of sample 1. Here, we assume that any external magnetic field and electric interaction between neighboring IJJs may be negligible. R_{fl} as R_{shunt} is also assumed to be ~75 mΩ under microwave injection. Thus, the equations to be solved are simplified [6]. Fig. 4 shows a typical numerically calculated dc *I–V* characteristic for i_r of 0.5 and f_{rf} of 10 GHz. This figure shows clear Shapiro steps, which seem to be the same as the experimental ones shown in Fig. 2. To make the behavior of both steps clearer, Fig. 5 shows a comparison between the i_r dependence of heights of the numerically obtained steps and the corresponding $P^{1/2}$

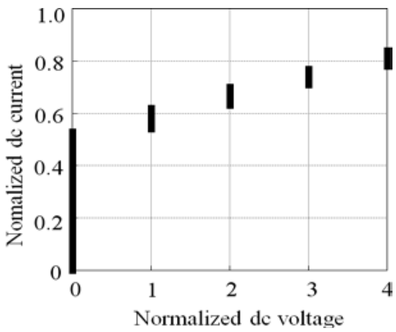

Fig. 4. The dc *I*–*V* characteristic calculated for sample 1 under injection of microwave of f_{rf} of 10 GHz.

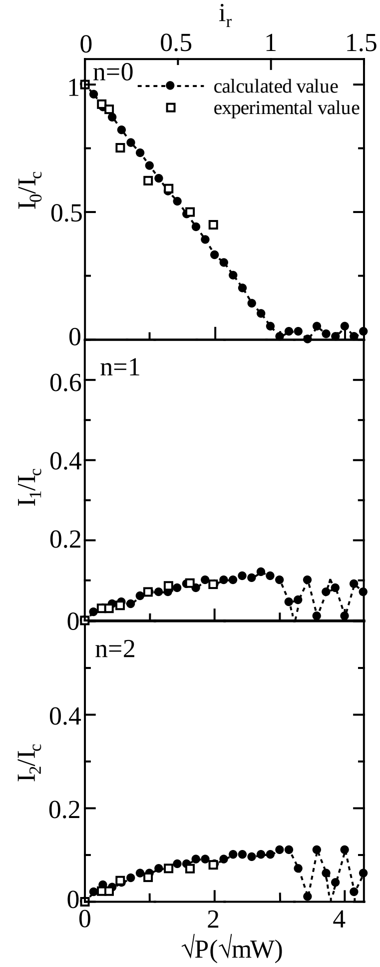

Fig. 5. Calculated and measured step heights as a function of i_r and $\sqrt{P}$, respectively, for f_{rf} of 10 GHz.

dependence of heights of the experimentally observed steps. As can be seen in this figure, close agreement between the observed and calculated values of step heights is obtained.

Moreover, we numerically calculate the *I*–*V* characteristics of the phase-locked JJs corresponding to the IJJs of sample 3 under microwave injection, using (1)–(6) on the sample conditions. In this case also, Shapiro steps are successfully observed as the steps of the IJJs are observed. Then, the influence of magnetic field on the Shapiro step response seems to be very weak.

From the above results, the experimentally observed steps may be regarded as Shapiro steps from the IJJs with Josephson-vortex flow resistivity corresponding to shunt resistivity.

V. Conclusion

We studied the response of intrinsic Josephson junctions in $(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$ (x = 0.15) at 4.2 K, which had high critical currents and Josephson-vortex flow resistivity, to injection of microwave at 2–20 GHz. Clear constant voltage steps were observed to satisfy the Josephson frequency-voltage relation on the current-voltage characteristic, although the plasma frequency was much higher than the microwave frequency and the junctions were not resistively shunted. Such steps were well reproduced by numerical calculations on Shapiro step response of Josephson junctions with shunt resistivity which was equal to the Josephson-vortex flow resistivity under microwave injection. The observed constant voltage steps were concluded to be Shapiro steps out of the IJJs with the Josephson-vortex flow resistivity. Such Shapiro steps with pronounced step heights may have advantages in applications over a wide temperature range.

References

[1] R. Kleiner, F. Steinmeyer, G. Kunkel, and P. Müller, "Intrinsic Josephson effects in $Bi_2Sr_2CaCu_2O_8$ single crystals," *Phys. Rev. Lett.*, vol. 68, pp. 2394–2397, Apr. 1992.

[2] G. Oya, N. Aoyama, A. Irie, S. Kishida, and H. Tokutaka, "Observation of Josephsonlike behavior in single-crystals $(Bi,Pb)_2Sr_2CaCu_2O_y$," *Jpn. J. Appl. Phys.*, vol. 31, pp. L829–L831, July 1992.

[3] K. Lee, W. Wang, I. Iguchi, M. Tachiki, K. Hirata, and T. Mochiku, "Josephson plasma emission from $Bi_2Sr_2CaCu_2O_y$ intrinsic junctions due to quasiparticle injection," *Phys. Rev. B* 61, pp. 3616–3619, Feb. 2000.

[4] H. B. Wang, P. H. Wu, and T. Yamashita, "Terahertz response of intrinsic Josephson junctions in high T_c superconductors," *Phys. Rev. Lett.*, vol. 87, pp. 107002-1–4, Sep. 2001.

[5] L. Ozyuzer, A. E. Koshelev, C. Kurter, N. Gopalsami, Q. Li, M. Tachiki, K. Kadowaki, T. Yamamoto, H. Minami, H. Yamaguchi, T. Tachiki, K. E. Gray, W.-K. Kwok, and U. Welp, "Emission of coherent THz radiation from superconductors," *Science*, vol. 318, pp. 1291–1293, Nov. 2007.

[6] M. Kitamura, A. Irie, and G. Oya, "Conditions for observing Shapiro steps in a $Bi_2Sr_2CaCu_2O_{8+\delta}$ high-T_c superconductor intrinsic Josephson junctions: Numerical calculations," *Phys. Rev. B* 76, pp. 064518-1–15, 2007.

[7] M. Kitamura, A. Irie, and G. Oya, "Shapiro steps in $Bi_2Sr_2CaCu_2O_{8+\delta}$ intrinsic Josephson junctions in magnetic field," *J. Appl. Phys.*, vol. 104, pp. 063905 1–13, Sep. 2008.

[8] R. L. Kautz and R. Monaco, "Survey of chaos in the rf-biased Josephson junction," *J. Appl. Phys*, vol.57, pp. 875–889, Feb. 1985.

[9] S. Rother, R. Kleiner, P. Müller, M. Darula, Y. Kasai, and Ke. Nakajima, "Far infrared response of intrinsic Josephson junctions," *Physica C*, vol. 341–348, pp. 1565–1566, 2000.

[10] Y.-J. Doh, J. Kim, K.-T. Kim, and H.-J. Lee, "Microwave-induced constant voltage steps in surface junctions of $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals," *Phys. Rev. B* 61, pp. R3834–R3837, Feb. (2000).

[11] G. Oya, T. Hashimoto, and A. Irie, "Microwave response of intrinsic Josephson junctions in $Bi_2Sr_2CaCu_2O_y$ single crystals at elevated temperatures," *Supercond. Sci. Technol.*,vol. 19, pp. S191–S194, 2006.

[12] H. Kaneoya, A. Irie, and G. Oya, "The behavior of subgap structures of intrinsic Josephson junctions in $(Bi,Pb)_2Sr_2CaCu_2O_y$ under magnetic field and microwave irradiation," *IEEE Trans. Appl. Supercond.*, vol. 19, pp. 195–198, June 2009.

[13] M. Machida and S. Sakai, "Unified theory for magnetic and electric field coupling in multistacked Josephson junctions," *Phys. Rev. B* 70, pp. 144520-1–6, 2004.

[14] A. Irie, M. Sakakibara, and G. Oya, "Growth and tunneling properties of $(Bi_{1-x}Pb_x)_2Sr_2CaCu_2O_y$ single crystals," *IEICE Trans. Electron.*, vol. E77-C, pp. 1191–1198, Aug. 1994.