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Design and Experimental Evaluation of SQIF Arrays with Linear Voltage Response

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***Abstract*—Differential circuits consisting of two series arrays of 10-junction parallel SQIFs were developed, designed and fabricated with 4.5 kA/cm^2 Nb HYPRES process. The differential voltage response evolution with applied magnetic field providing opposite frustration of the serial arrays was analyzed in detail. Linear differential response with amplitude as high as 22 mV was observed for the serial arrays of 108 parallel SQIFs. It was shown that the response linearity is kept within some range of the applied frustrating magnetic field.**

***Index Terms*—Josephson junctions, SQIF, differential circuit, voltage response, high linearity.**

I. Introduction

Superconducting Quantum Interference Filters (SQIF) of both parallel and series types were first introduced in 2001 [1]-[3] and created a considerable interest. Nonperiodic voltage response with a single sharp peak at zero magnetic field as well as the improved protection from electromagnetic noise environment make SQIF circuits very attractive for applications in sensitive magnetometry [4]-[8]. RF signal amplification and RF signal mixing using SQIFs were also studied theoretically and experimentally [9]-[13]. The SQIF circuits were successfully used in the design of SFQ-pulse driver to generate output pulses with higher amplitudes [14]-[17]. One should also mention research of higher complexity SQIF structures [18]-[20], two-dimensional SQIF arrays [21], parallel SQIFs consisting of Josephson junctions with unconventional current-phase relation [22], [23] and studies of oscillation linewidth and noise characteristics of parallel SQIF [24]. There are good reasons to consider SQIF arrays as promising candidates for synthesis of high linearity array structures for the use as high performance amplifiers in gigahertz frequency range where external feedback loop can not be implemented.

Recently we proposed Josephson-junction structures capable of providing a SQIF-like high linearity voltage response [25]-[26]. These structures are based on the use of a differential scheme of two magnetically frustrated parallel SQIFs (see Fig. 1) with special distribution $a(x)$ of the cell areas along the array and critical current biasing $I_b = (I_c)_{SQIF}$. Furthermore, we developed this circuit into a differential serial–parallel structure and resolved some design issues to increase performance of such arrays [27]-[29]. In this paper we present results of experimental evaluation of the integrated circuits containing differentially-connected series arrays of parallel 10-junction SQIFs.

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II. Theory

At vanishing inductances between Josephson junctions in parallel SQIF one can use analytical relation for the parallel SQIF voltage response [1], [2]:

$$V(B) = V_c\sqrt{(I_b/I_c)^2 - |S_K(B)|^2}\ , \tag{1}$$

where B is magnetic field, and $S_K(B)$ is a structural factor:

$$S_K(B) \ = \ \frac{1}{K}\sum_{k=1}^{K}\exp\left(i\frac{2\pi}{\Phi_0}B\sum_{m=1}^{k-1}a_m \right), \tag{2}$$

I_b – bias current, I_c – total critical current of SQIF, K – number of Josephson junctions, a_m – effective area of the m-th interferometer cell. At sufficiently high number K, one can use integration instead of summation, and (2) can be transformed as follows:

$$S(B) \ = \ \frac{1}{L}\int_0^L dz\cdot\exp\left(i\frac{2\pi}{\Phi_0}B\int_0^z a(x)dx \right). \tag{3}$$

We need to find such a special distribution $a(x)$ of the interferometer cell areas along the SQIF-structure $(0 < x < L)$ which makes the differential circuit voltage response

$$\Delta V(B) \ = \ V(B+\delta B) - V(B-\delta B) \tag{4}$$

close to a linear law

$$\Delta V(B) \ = \ k\cdot B \tag{5}$$

in a signal region $-\alpha\cdot\delta B < B < \alpha\cdot\delta B$, where $\alpha \le 1$ and δB is magnetic frustration of the SQIFs.

Equations (1)-(5) allow deriving a master equation and formulating the minimization problem for the obtained functional. By solving numerically this problem, one can find

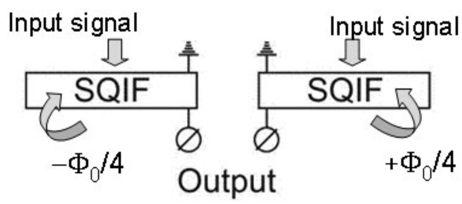

Fig. 1. Differential scheme of two parallel SQIFs frustrated by $\Phi_0/2$.

the optimal distribution $a(x)$. In order to do this, one should set an initial approximation and then use an iterative minimization algorithm. We succeeded in solving this problem at critical biasing of the SQIFs $I_b = (I_c)_{SQIF}$ and came to best distribution for the effective cell areas as follows:

$$a(x)/a_\Sigma = 1.2 - 0.48\sin^3(\pi x) \; . \qquad (6)$$

Here x is a coordinate of cell, and a_Σ is a total area of the cells. If a control current line is used to apply magnetic flux to the SQIF cells, the coefficients of mutual inductances between the line and the SQIF cells play a role of the effective cell areas.

In case of finite inductances l of the interferometer cells, the SQIF response $V(B)$ has to be calculated numerically, e.g. using PSCAN routine [30].

One can show that the most optimal solution of this minimization problem leads to a parabolic shape of the SQIF peak voltage response. In this case, the differential voltage response formed by the parabolic peaks of the SQIF responses is linear regardless of value δB of the frustrating magnetic field.

In fact, if the SQIF response peak $V(B)$ is fully symmetric, the linearity condition expressed by equation (5) can be written as follows:

$$\frac{d(\Delta V(B))}{dB} \equiv \frac{dV(\delta B + B)}{dB} + \frac{dV(\delta B - B)}{dB} = k = const \qquad (7)$$

In case of parabolic function

$$V(B) = bB - aB^2 + c \qquad (8)$$

with derivative

$$\frac{dV(B)}{dB} = b - 2aB\,, \qquad (9)$$

(7) gives a linear differential voltage response with the slope factor k depending on value δB of the frustrating magnetic field:

$$\frac{d\Delta V(B)}{dB} = 2b - 4a\delta B = k = const\,. \qquad (10)$$

Equation (10) evidences that differential voltage response $\Delta V(B)$ is linear regardless of the value δB, when parabolic parts of the SQIF responses are subtracted.

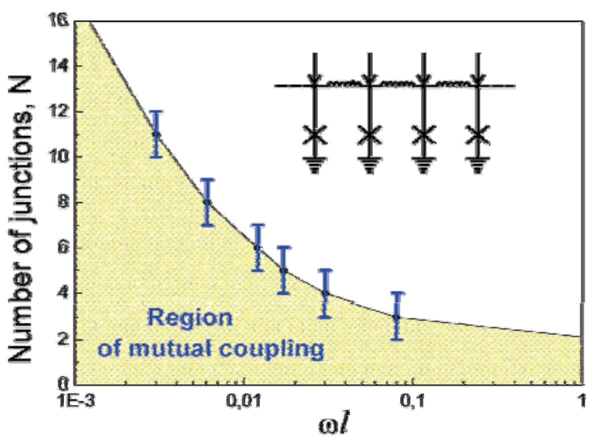

Fig. 2. Josephson junction interaction radius for parallel array versus normalized coupling impedance ω*l*. Here ω is normalized by characteristic Josephson frequency.

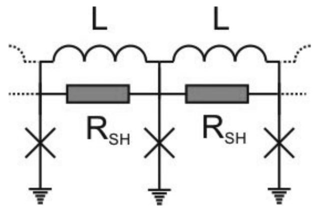

Fig. 3. The coupling inductances are shunted by low-ohmic resistors to decrease coupling impedance at high frequency and hence to extend the interaction radius up to size of 10-junction SQIF.

Formula (1) for parallel SQIF voltage response becomes inapplicable in the case of finite values of coupling inductances l between Josephson junctions. In this case, the voltage response is formed by a limited number of Josephson junctions contained within the interaction radius [26]. The interaction radius decreases with inductance and frequency as shown in Fig. 2. Decreasing the normalized inductance value $l = 2\pi I_C L/\Phi_0$ down to 0.5 and shunting the coupling inductances by low-ohmic resistors $r_S \equiv R_{SH}/R_N \approx 0.6$ to decrease coupling impedance at high frequency $\Omega > R_{SH}/L$, one can extend the interaction radius up to the size of a 10-junction SQIF. Voltage response of such parallel SQIF is very close to the one given by (1) and hence the obtained effective cell area distribution (6) is quite applicable.

III. Experimental Evaluation

Integrated circuits with differentially connected series arrays of parallel 10-junction SQIFs were designed and fabricated using HYPRES 4.5 kA/cm^2 Nb process [31]. The coupling inductances between Josephson junctions in the parallel SQIFs were reduced down to normalized value $l \approx 0.5$ at Josephson-junction critical current $I_C = 0.125$ mA and were shunted by resistors $R_{SH} \approx 0.8$ Ohm to extend radius of the junction interactions up to the entire SQIF size. The resistively shunted tunnel Josephson junctions were characterized by normal resistance $R_N \approx 1.6$ Ohm and McCumber parameter $\beta_C \approx 0.2$. The control-current strip line intended for the magnetic flux application had variable mutual inductances between the line and the SQIF cells to provide distribution of the effective cell areas in accordance with (6). Variable mutual inductances

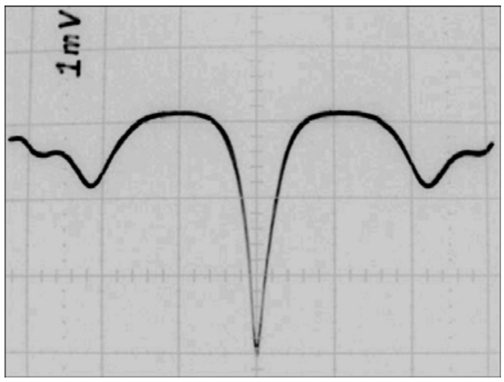

Fig. 4. Oscilloscope trace shows voltage responses of the serial array of 56 parallel 10-junction SQIFs. Mutual inductances between SQIF cells and control line intended for magnetic field application are set in compliance with eq. (9). Bias current is a bit more than critical current of the SQIF. Vertical scale is 1 mV/div. Horizontal scale is 10 mA/div (about $4\Phi_0$/div).

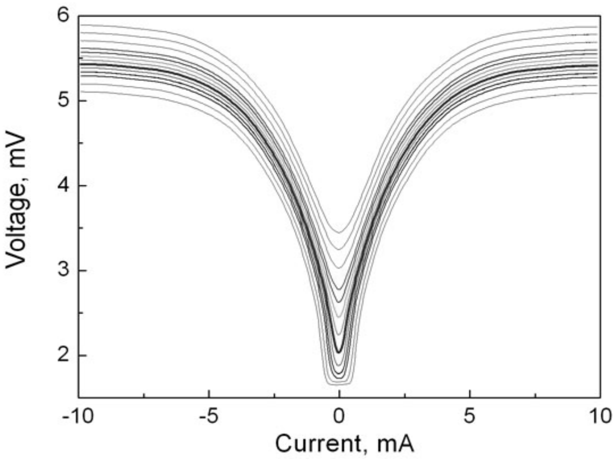

Fig. 5. A series of voltage responses of the serial array of 56 parallel SQIFs with increase in bias current from $I_b \leq (I_c)_{SQIF}$ (lower curve) to $I_b \approx 1.06 \cdot (I_c)_{SQIF}$. Each parallel SQIF consists of 10 Josephson junctions.

were realized by two different methods producing similar results. In the first method, a uniform control strip line lays over the different-of-length strip sections forming inductances of the SQIF cells. In the second method, we implemented a variable-width control strip line lying over the equal strip sections forming cell inductances. In addition, we used individual sections of double ground planes for each parallel SQIF in the serial arrays to eliminate shunting effect of the stray capacitances which are characteristic for the standard circuit designs with two superconducting ground planes [28].

Fig. 4 presents the measured typical voltage response of the serial array of 56 parallel SQIFs biased by current I_b which slightly exceeds the SQIF critical current $(I_c)_{SQIF} = 10 \cdot I_C$. Each parallel SQIF consists of 10 Josephson junctions and conforms the effective cell area distribution given by (6). Fig. 5 shows a series of the voltage responses with increasing bias current from $I_b \leq (I_c)_{SQIF}$ to $I_b \approx 1.06 \cdot (I_c)_{SQIF}$.

We also fabricated and measured the two-times larger differential arrays. Voltage responses of two differential serial arrays are presented in Fig. 6. Each array contains 108 parallel 10-juction SQIFs connected in series via very low resistors (~0.01 Ohm). While no magnetic field is applied to oppositely frustrate the arrays, a zero differential response is observed. When the frustrating magnetic field δB is applied, this circuit shows the bipolar high-amplitude differential voltage response with linear central part as shown in Fig. 7. The amplitude of the bipolar voltage response is as high as 22 mV. A series of the differential voltage responses with increase in the frustrating magnetic field δB is shown in Fig. 8. It is seen that linear character of the responses takes place within some range of δB. This fact unambiguously proves the parabolic shape of the SQIF response parts that forming these linear differential array responses.

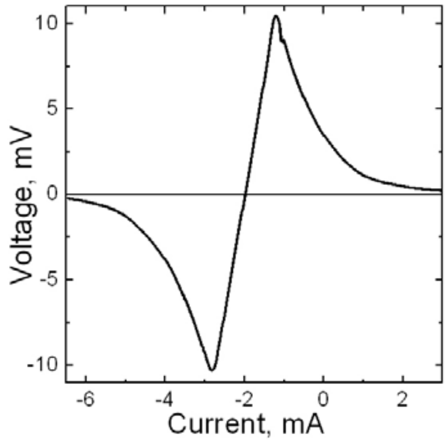

Fig. 7. Differential voltage responses of two serial arrays of 108 parallel 10-junction SQIFs. The peak-to-peak response amplitude is about 22 mV.

IV. Discussion

The experimentally measured voltage responses show a highly linear shape. In order to characterize the designed circuits quantitatively in terms of nonlinear distortion, a two-tone analysis needs to be done.

The theoretical analysis [26] of differential circuits at vanishing inductances between Josephson junctions in parallel SQIFs shows the increase in the differential response linearity with number of Josephson junctions in the SQIFs. However, in order to cover more junctions within the interaction radius and therefore contributing to the parallel SQIF voltage response,

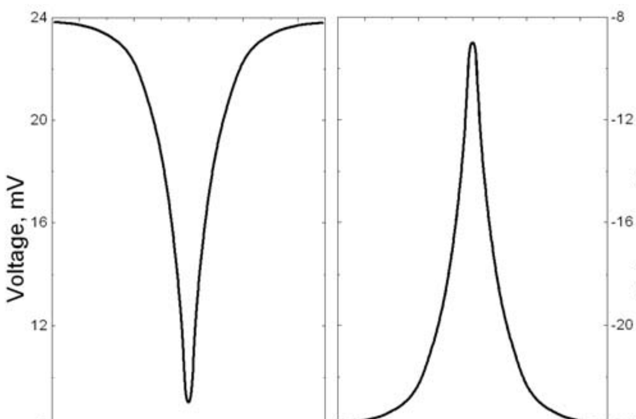

Fig. 6. Voltage responses of two differentially connected arrays each consisting of 108 parallel 10-junction SQIFs connected in series via very low resistors (~ 0.01 Ohm). The response amplitude is about 15 mV.

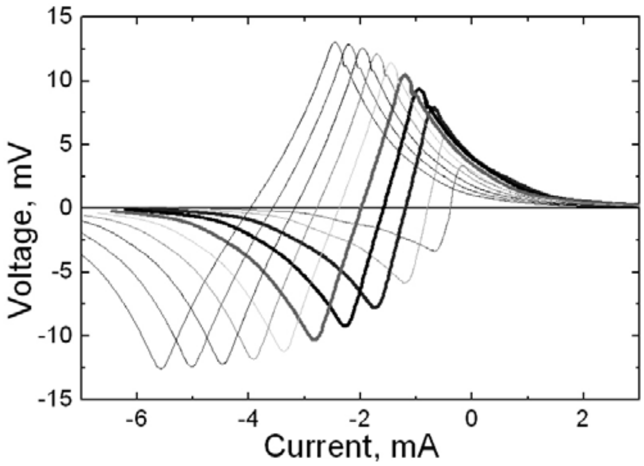

Fig. 8. A series of the differential voltage responses of two series arrays each consisting of 108 parallel 10-junction SQIFs with increasing frustrating magnetic field δ*B*.

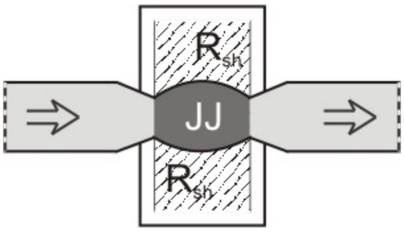

Fig. 9. Sketch of resistively shunted long tunnel Nb Josephson junction as the ultimate parallel SQIF structure. The control current strip line applying magnetic field to the distributed junction lies on top. The junction shape (dark area) should repeat the shape of the strip line with width varying in accordance with (6) to provide the required magnetic field distribution along the long junction.

one should decrease coupling inductances between the junctions. This leads to the ultimate structure which is a distributed overdamped junction with the length limited by the interaction radius. In practice, this junction can be designed as a shunted resistively distributed tunnel junction shown schematically in Fig. 9. The control current strip line inducing magnetic field to the distributed junction lies above. The junction shape should repeat the shape of the strip line with the width varying in accordance with (6) to provide the required magnetic field distribution along the junction.

One can also suggest to design and integrate a differential circuit with two parallel SQIFs or two distributed Josephson junctions into one cell providing a linear voltage response. Fig. 10 presents a possible schematic of the cell aimed at high-frequency applications. In this cell, two parallel SQIFs (or two distributed junctions) are connected in series for dc biasing but differentially for high frequency signals.

The integrated cells can be connected in series to increase dynamic range and output signal amplitude. Moreover, such a serial array of the cells each providing highly linear voltage response could be used to design active electrically small antennas [32].

V. Conclusion

Differential circuits consisting of two serial arrays of the specially modified 10-junction parallel SQIFs were designed, fabricated and tested. The differential voltage response dependence on the applied magnetic field providing opposite

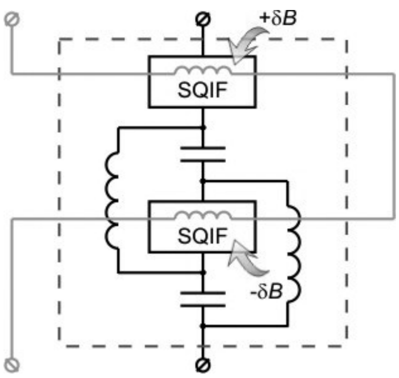

Fig. 10. Elementary cell with two SQIFs biased differentially by dc current and dc magnetic flux and connected serially for input and output high frequency signals (the input signal circuit is not shown here). The compact arrangement of the SQIFs enables easy magnetic signal input for both SQIFs simultaneously.

frustration of the serial arrays was analyzed in detail. Linear differential response with amplitude as high as 22 mV was observed for the serial arrays of 108 parallel SQIFs. It was found that the response linearity can be sustained within some range of the frustrating magnetic field.

Further progress in higher linearity in SQIF array circuits can be obtained with implementation of the specially designed distributed Josephson junctions as well as with the use of the suggested integrated cells with linear response.

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