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Effect of LC-shunting on the IV-characteristics of a Josephson Junction under Microwave Radiation

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***Abstract*—We study the resonance features of the coupled system of Josephson junctions shunted by the LC-elements under electromagnetic irradiation. A strong effect of the external radiation on the IV-characteristics and voltage-time dependence is demonstrated. Crucial changes are found at the resonance condition when radiation frequency coincides with the Josephson and resonance circuit frequencies. It changes the amplitude dependence of the Shapiro step width. The optimized LC shunt leads to the increased step height for steps on the resonance branch of IV-characteristics at low amplitudes. The shunting of the Josephson junctions provides an extended range using the same microwave source, because Shapiro step demonstrates the first Bessel maximum at a much smaller power of radiation in comparison to the case of unshunted Josephson junctions. These features of Shapiro step on the resonance branch might be interesting for quantum metrology.**

I. Introduction

The high-T_c superconductor $Bi_2Sr_2CaCu_2O_{8+x}$ is equivalent to a stack of tunnel Josephson junctions (JJs) [1] called intrinsic Josephson junctions. The system of intrinsic JJs is a promising object in HTSC electronics that has been extensively studied in recent years [2], [3], [4], [5]. One of the effective methods for affecting the Josephson junction is its shunting by LCR elements [6], [7], [8], [9], [10]. In particular, shunting leads to the synchronization of oscillations of the superconducting current in an array of JJs. JJs, together with the LCR elements, form an oscillatory circuit. When the Josephson frequency ω_J becomes equal to the natural frequency of the circuit ω_{rc}, oscillations in JJs are tuned to this frequency. This resonance is manifested in the IV-characteristics (IVC) in the form of various features such as steps [11], [12], humps or dips [13], [14]. In particular, the existence of such resonance features in the IVC in various systems of JJs with a resonance circuit was reported in a number of experimental and theoretical works (see [15], [16], and references therein). A peak in the intensity of coherent electromagnetic radiation from a two–dimensional system of JJs based on Nb/Al/AlOx/Nb was detected in [17] at the synchronization of oscillations in different JJs, which is caused by this resonance. The considered system has an interesting potential for applications in quantum metrology [18].

An interesting problem concerns the effect of electromagnetic radiation on this system of JJs shunted by LCR elements. First, in general case (without shunting) external radiation produces an additional superconducting current. It leads to the appearance of the Shapiro steps (SS) and their subharmonics in the IV-characteristics, which position and width depend on radiation frequency ω_R and amplitude A. Second, a double resonance can be realized at the conditions $\omega_J = (m/n)\omega_{rc} = (p/q)\omega_R$, where n, m, p and q are integer. Third, the triplet resonance reveals in the stack of intrinsic JJs when four frequencies are coincide: Josephson's, radiation's, resonance circuit's and double longitudinal plasma wave's. The properties of SS in this case are not investigated enough yet. Particularly, the question about amplitude dependence of SS width ΔI on an rc-branch is steel under discussion [19].

In this paper, we study the influence of the external electromagnetic radiation on the phase dynamics of coupled system of JJs shunted by LC-elements. We simulate the IV-characteristics and time dependence of voltage in JJ and present results of detailed investigation of the rc-branch. The variation of IV-characteristics and the amplitude dependence of SS width ΔI in the system of JJs under radiation with SS on the rc-branch are discussed.

II. Models and Methods

Let us consider the system, presented in Fig. 1. In normalized units the system of equations, describing this electric scheme can be written in the form [20]

$$\begin{cases} \dfrac{\partial \varphi_l}{\partial t} = V_l - \alpha(V_{l+1} + V_{l-1} - 2V_l) \\ \dfrac{\partial V_l}{\partial t} = I + A\sin\omega_R t - \sin\varphi_l - \beta\dfrac{\partial \varphi_l}{\partial t} - C\dfrac{\partial u_c}{\partial t} \\ \dfrac{\partial^2 u_c}{\partial t^2} = \dfrac{1}{} \end{cases}$$

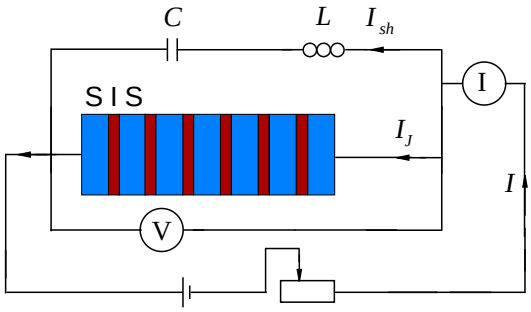

Fig. 1. (Color online) Schema of a stack of JJs with LC shunting elements and the Josephson and the shunt current

time - to the inverse plasma frequency $\omega_p = \sqrt{\frac{2eI_c}{C_j\hbar}}$, voltages V_l and u_c are normalized to $V_0 = \frac{\hbar\omega_p}{2e}$; shunt capacitance C- to the capacitance C_j of the JJ, and shunt inductance L - to $(C_j\omega_p^2)^{-1}$. In the system of equations (1) we introduce a dissipation parameter $\beta = \frac{1}{R_j}\sqrt{\frac{\hbar}{2eI_cC_j}} = \frac{1}{\sqrt{\beta_c}}$ with β_c as McCumber parameter.

We study the phase dynamics of the system based on the eq.(1) and discuss the influence of LC shunting on IV-characteristics and resonance features in this system. We note that the JJs together with LC-elements form a resonance circuit with its eigenfrequency

$$\omega_{rc} = \sqrt{\frac{1+NC}{LC}}, \qquad (2)$$

where N is the number of JJs in the stack.

We have found that the LC shunting leads to the step structure in the oneloop IV-characteristics, when the value of Josephson frequency approaches to the eigenfrequency of the LC resonance circuit [13], [14]. The location of the step depends on parameters of this LC circuit [20].

III. Shapiro step on the RC-branch of single JJ

Fig.2 demonstrates time dependence of voltage and IV-characteristics of single JJ under external radiation with frequency $\omega_R = 3$ and amplitude $A = 0.5$. Fig.2(a) shows IV-characteristics of JJ without shunting. Filled arrows indicate the direction of current sweeping. Time dependence has been calculated for the area labeled by dashed line on the IV-characteristics. Inset enlarge this area with the main SS at $V = 3$. In Fig.2(c) we demonstrate time dependence of voltage. The decrease in voltage oscillations in the numerical calculations corresponds to the SS on IV-characteristics. Left inset enlarge oscillations before the SS. The oscillations of voltage modulated by external force $Asin(\omega_R t)$ in this region. Right inset presents oscillations of voltage in the region which correspond to the SS.

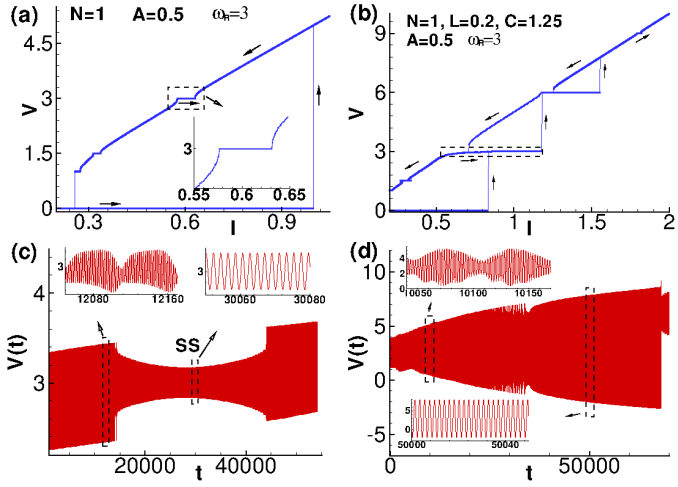

Fig. 2. (Color online) Time dependence of voltage and IV-characteristics of JJ under external radiation with frequency $\omega_R = 3$ and amplitude $A = 0.5$. (a) The IV-characteristics of JJ without shunting, inset enlarges the SS. (c) Time dependence of voltage for the labeled area on IV-characteristics. Inset enlarges oscillations of voltage which correspond to before the SS (left) and the SS (right) regions. (b) The IV-characteristics of JJ shunted by $L = 0.2$ and $C = 1.25$ elements. (d) Time dependence of voltage of rc-branch. Insets enlarge the oscillations of voltage which correspond to the regions before the SS (upper) and SS (down).

Fig.2(b) demonstrates the IV-characteristics of a single JJ shunted by $L = 0.2$ and $C = 1.25$ elements. The frequency of resonance circuit $\omega_{rc} = 3$ is equal to radiation frequency ω_R. Shapiro step located on the rc-branch labeled by dashed line. In Fig.2(d) we show time dependence of voltage calculated for the labeled area in IV-characteristics. The amplitude of voltage increases on the rc-branch because of the resonance $\omega_{rc} = \omega_J$. Upper inset enlarges the modulated oscillations in the rc-branch region before the SS. Lower inset enlarges the oscillations corresponded to the SS. Here the effect of frequency locking takes place. The amplitude of voltage oscillations is increasing along the SS. We consider that resonance between ω_{rc} and ω_J is a reason of this effect.

IV. Stack with N=10 JJ under external radiation

Fig.3 shows the IV-characteristics of a stack with 10 intrinsic JJs shunted by $L = 48$ and $C = 0.002$ elements under external electromagnetic radiation with frequency $\omega_R = 3.2596$ and amplitude $A = 0.5$. Arrows indicate the direction of current sweeping. We see a big rc-branch at $V = 32.596$ and the second harmonic at $V = 65.110$. Irradiation leads to the appearance of the Shapiro steps on rc-branch. Inset enlarges the rc-branch demonstrating this Shapiro step on it.

Fig.4 demonstrates the dependence of SS width ΔI on the amplitude of external radiation. The results marked by SS show the A-dependence of ΔI_{SS} in the case without shunting. The results marked by the SS-rc show the same dependence when SS is on the resonance branch of the system with 10 JJs. We see that the amplitude dependence of the Shapiro step width are crucially changed in the case of shunting. At small A, ΔI_{SS-rc} is larger in the case of shunting than ΔI_{SS} for the system of JJs without shunting. The period of the Bessel function is decreased in comparison with the case of JJs without shunting.

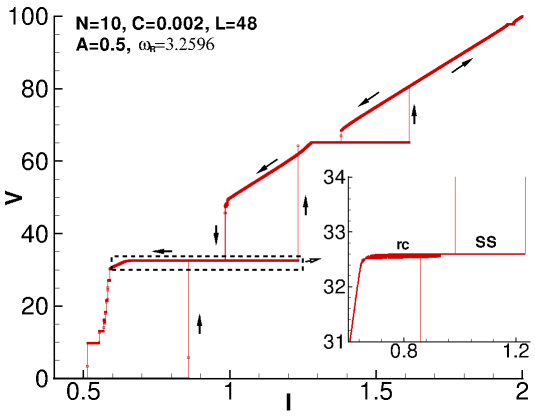

Fig. 3. (Color online) The IV-characteristics of the stack with 10 JJ under external radiation with frequency $\omega_R = 3.2596$ and amplitude $A = 0.5$. Inset enlarges the rc-branch demonstrating Shapiro step on it.

We note that the amplitude dependence of ΔI in shunted case was calculated up to $A = 27$.

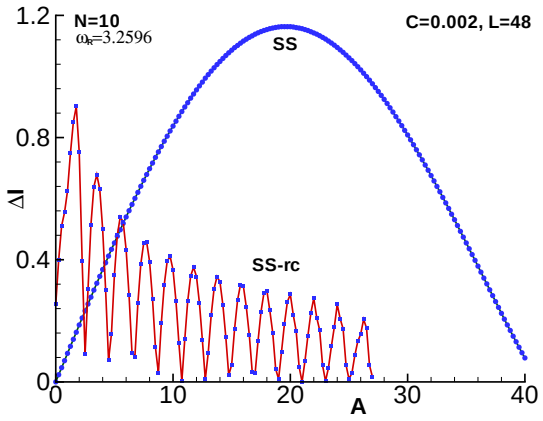

Fig. 4. (Color online) The amplitude dependence of the first SS ΔI on the resonance branch (SS-rc) at $\omega_R = 3.2596$, compared to the same dependence without shunting (SS).

V. Conclusion

We have studied the resonance features of coupled system of Josephson junctions shunted by LC-elements under electromagnetic radiation. A strong effect of the external radiation on voltage-time dependence is demonstrated. We have shown the change in the Bessel behaviour at the resonance condition $\omega_R = \omega_J = \omega_{rc}$. The dependence of ΔI of Shapiro step on the amplitude of external radiation is essentially changed.

Acknowledgment

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