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Modelling and Comparison of In-Field Critical Current Density Anisotropy in High Temperature Superconducting (HTS) Coated Conductors

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***Abstract*—The development of high temperature superconducting (HTS) wires is now at a stage where long lengths of high quality are available commercially, and of these, (RE)BCO coated conductors show the most promise for practical applications. One of the most crucial aspects of coil and device modelling is providing accurate data for the anisotropy of the critical current density $J_c(B, \theta)$ of the superconductor. In this paper, the in-field critical current density characteristics $J_c(B, \theta)$ of two commercial HTS coated conductor samples are measured experimentally, and based on this data, an engineering formula is introduced to represent this electromagnetic behaviour as the input data for numerical modelling. However, due to the complex nature of this behaviour and the large number of variables involved, the computational speed of the model can be extremely slow. Therefore, a two-variable direct interpolation method is introduced, which completely avoids any complex data fitting for $J_c(B, \theta)$ and expresses the anisotropic behaviour in the model directly and accurately with a significant improvement in computational speed. The two techniques are validated and compared using numerical models based on the H-formulation by calculating the self-field and in-field DC critical currents and the AC loss for a single coated conductor.**

***Index Terms*—Ac loss, critical current density (superconductivity), finite element analysis, high-temperature superconductors, numerical analysis**

I. Introduction

It is a crucial aspect of superconducting coil and device modelling to provide accurate data for the anisotropy or angular dependence of the critical current density $J_c(B, \theta)$ for the superconductor. If this behaviour is known in detail, it is possible to accurately predict the critical current and AC loss of coils and other devices. There are two methods – data fitting and interpolation – to include the experimental data of the anisotropic properties of a superconducting wire.

We are currently investigating the design of an all-superconducting axial flux-type electric machine using HTS materials in both bulk and wire forms [1] and evaluating the performance of test HTS coils for this. As a starting point, this involves carrying out the measurement of the in-field critical current density properties of commercial HTS coated conductor. The experimental data for the modelling in this work comes from two short samples from longer spools of wire (approximately 20 m) manufactured by SuperPower and used to wind the test coils. Firstly, the basic properties and trends of the experimental data for these two samples are described. Based on these two data sets, we developed a method for data fitting and interpolation separately. For the data fitting, we develop an engineering formula to reproduce the measurements accurately from 0 T to 0.7 T. For the interpolation, we develop a two-variable direct interpolation to include $J_c(B, \theta)$ completely and directly. This is a simple and direct method, which avoids any complex data fitting and mathematical calculations.

To validate these models, numerical analyses are carried out using the ***H***-formulation implemented in the commercial software package, Comsol Multiphysics 4.3a. The numerical results for the self-field and in-field DC critical current are compared in detail, and a comparison of the computational times required for each method is made.

II. Experimental Data Analysis

Both samples are SuperPower's SCS4050-AP coated conductor and labelled as SP1 and SP2. To measure the critical current, each sample was cooled slowly to 77 K. A voltage criterion of E_0 = 1 μV/cm is used to define the critical current. The experimental results for SP1 and SP2 are shown in Figs. 1 and 2, respectively. For both samples, the magnitude of the applied field was varied from 0 to 0.7 T in 0.05-0.1 T increments. The symbols in Figs. 1 and 2 represent the measured experimental results and the solid lines represent the numerical data fitting/interpolation.

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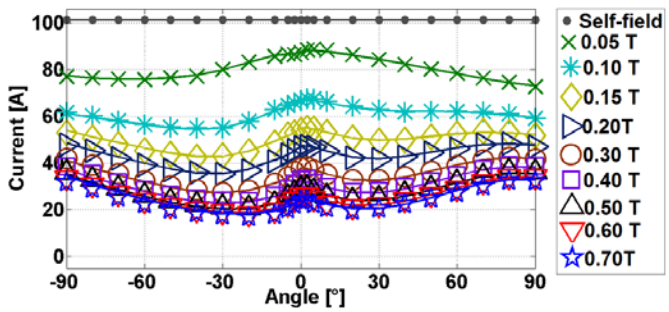

Fig. 1. Comparison of the experimental (symbols) and numerical data fitting (solid lines) for the angular, in-field dependence of the critical current density $J_c(B, \theta)$ for sample SP1.

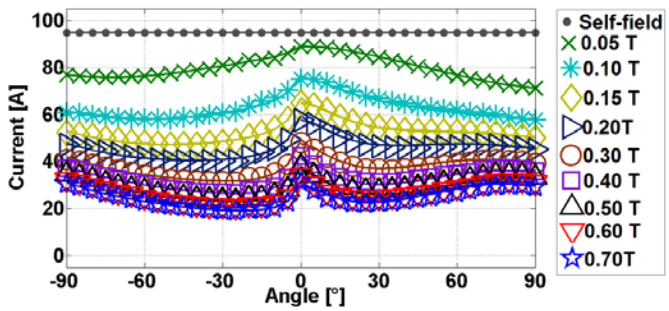

Fig. 2. Comparison of the experimental (symbols) and numerical data fitting (solid lines) for the angular, in-field dependence of the critical current density $J_c(B, \theta)$ for sample SP2.

Considering these similar trends in the two samples, an engineering formula can be developed for data fitting to input these data into the numerical model. For the data fitting, the basic form of the equation used is described by (1).

$$I_c(B,\theta) = I_{c0} / (1 + B\sqrt{v(B,\theta)^2 \cos^2\theta + u(B,\boldsymbol{\theta})^2 \sin^2\theta} / B_0)^{\beta} \quad (1)$$

where I_{c0} is the self-field critical current, B_0 and β are constants that depend on the material. Coefficients u and v are functions of the applied field magnitude B and field angle θ, and u and v are described by (2) and (3), respectively.

$$u(B,\theta) = b(B)\cos\theta + c(B)\theta + d(B) \quad (2)$$

$$v(B,\theta)^2 = a(B)^2, \text{ when } \theta \geq 0$$

$$v(B,\theta)^2 = \left| f(B)(\theta - \theta_0)\exp(g(B)\theta) \right|, \text{ when } \theta < 0 \quad (3)$$

where a-d, f, and g are the functions of the applied field magnitude B, and θ_0 is a constant, which again depends on the material.

For SP1, we find B_0 = 0.319, β = 2.405, I_{c0} = 101.4, and θ_0 = 5. Because of the asymmetric tape behaviour, the functions of a-d, f, g should be considered separately when $\theta \geq 0$ and $\theta < 0$.

For $\theta \geq 0$:

$$a(B) = 0.7174\exp(-0.9624B) - 1.567\exp(-32.3B) \quad (4)$$

$$b(B) = -3.606\exp(-1.001B) + 5.353\exp(-12.93B) \quad (5)$$

$$c(B) = -3.509\exp(-0.981B) + 5.818\exp(-13.41B) \quad (6)$$

$$d(B) = 6.139\exp(-1.002B) - 8.715\exp(-13.96B) \quad (7)$$

For $\theta < 0$:

$$b(B) = 5.087\exp(-1.372B) - 21.69\exp(-37.35B) \quad (8)$$

$$c(B) = -5.593\exp(-1.349B) + 20.09\exp(-34.71B) \quad (9)$$

$$d(B) = -9.557\exp(-1.366B) + 33.77\exp(-36.22B) \quad (10)$$

$$f(B) = 6.286\exp(-2.149B) - 15.94\exp(-29.13B) \quad (11)$$

$$g(B) = 8.19\exp(-1.81B) + 1.004\exp(1.519B) \quad (12)$$

Based on equations (1)-(12), the data is fit for SP1 as shown in Fig. 1. It can be seen that our data fitting describes the experiment $J_c(B, \theta)$ data very well from self-field up to 0.7 T.

For SP2, we find B_0 = 0.5, β = 1.446, I_{c0} = 94.7, and θ_0 = 2.7.

For $\theta \geq 0$:

$$a(B) = 1.15\exp(-0.4108B) - 1.831\exp(-19.68B) \quad (13)$$

$$b(B) = -1.526\exp(1.625B) + 5.557\ln(B + 10^{-5}) + 28.27\exp(-9.14B) \quad (14)$$

$$c(B) = -8.954\exp(-0.4397B) + 15.33\exp(-10.36B) \quad (15)$$

$$d(B) = 15.61\exp(-0.4722B) - 23.38\exp(-10.77B) \quad (16)$$

For $\theta < 0$:

$$b(B) = 4.997\exp(-0.596B) - 12.17\exp(-11.24B) \quad (17)$$

$$c(B) = -6.042\exp(-0.547B) + 13.88\exp(-12.63B) \quad (18)$$

$$d(B) = -10.97\exp(-0.5825B) + 22.62\exp(-13.62B) \quad (19)$$

$$f(B) = 26.25\exp(-0.9123B) - 41.28\exp(-13.54B) \quad (20)$$

$$g(B) = 4.046\sin(1.882B) + 54.94B\exp(-8.025B) \quad (21)$$

Based on equations (13)-(21), the data is fit for SP2 as shown in Fig. 2, which again matches very well.

Alternatively, an interpolation function can be used to describe the relationship between these coefficients and applied field B. The two-variable direct interpolation method proposed here is a simpler and more direct method, similar to a look-up table. All of the experimental data can be input as a single function, with two input variables, B and θ, and one output variable, the critical current density, J_c, using a direct interpolation, which is available in Comsol. This significantly simplifies the process, and as shown later, dramatically improves the computational time required.

Both the data fitting and two-variable direct interpolation are valid and accurate methods to include the experimental data in the model. However, accurate data fitting of complex $J_c(B, \theta)$ behaviour like this needs complicated mathematical functions.

III. Modelling and Simulation

Now that the two methods to include the experimental data have been defined, we should consider the accuracy of and computational time required when used in a numerical model. A 2D infinitely long model [2], [3] is used to investigate the electromagnetic properties of a single tape, based on the ***H***-formulation [2]-[5]. The width of the tape is 4 mm and the thickness of the superconducting layer is 1 μm. The tape is surrounded by an air sub-domain. The real thickness and width are modelled by the finite element method using Comsol. A mapped mesh is used in the superconducting layer to decrease the number of mesh elements [6] and a free triangular mesh is used in the air sub-domain. Linear, curl-conforming elements are employed for the entire model.

IV. Result and Discussion

A comparison of the numerical simulation results for the self-field and in-field DC critical currents for the short samples SP1 and SP2 using the data fitting and two-variable direct interpolation methods are shown in Figs. 3 and 4, respectively.

Based on Figs. 3 and 4, it can be seen that our simulation results by data fitting and two-variable direct interpolation are consistent with the experimental results, so their accuracy is validated.

To assess the computational speed of the two methods, the time required to solve each set of model parameters for the data fitting and two-variable direct interpolation methods is shown in Table I. Table I shows the computational time for the models for the short sample SP1 at applied field angles of 0°, ±30°, and ±90°. Very similar results were observed for SP2, the results for which have been omitted here. DF represents the data fitting method; INT represents the two-variable direct interpolation method.

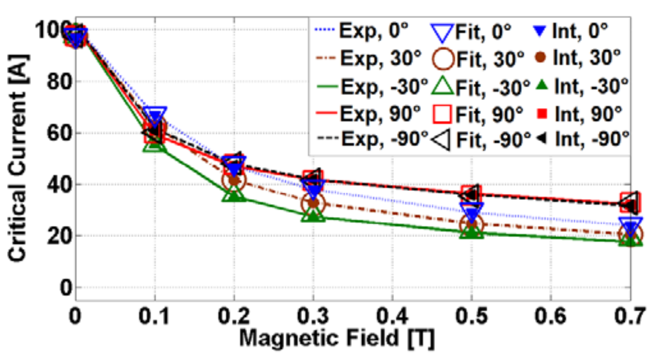

Fig. 3.Comparision of experimental result (lines), simulation result with data fitting (open symbols) and simulation result with two-variable interpolation (closed symbols) for short samples 1.

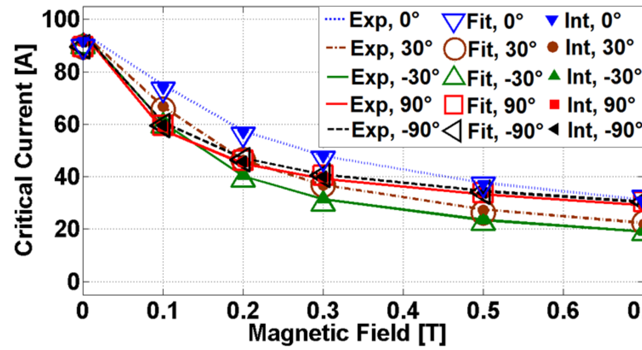

Fig. 4.Comparision of experimental result (lines), simulation result with data fitting (open symbols) and simulation result with two-variable interpolation (closed symbols) for short samples 2.

From Table I, it can be seen that the computational time for the two-variable direct interpolation are all of the same order for all cases and vary between 300 s to 679 s. For the data fitting method, the computational time varies greatly depending on the angle and magnitude of the applied field, but is significantly slower than the interpolation method, varying from around five times slower (±90°) up to over 100 times slower (0°). Hence, the two-variable direct interpolation is not only much faster, but much more consistent in terms of model convergence and solver time.

TABLE I
Computational Time Required to Calculate the In-Field DC Critical Current Using the Data Fitting and Two- Variable Direct Interpolation Methods for Short Sample SP1 at Different Applied Field Angles. All Values are Given in Units of Seconds (s).

Applied Field	0°		-30°		30°	
	DF	INT	DF	INT	DF	INT
0 T	1958	344	2245	375	2145	368
0.1 T	36878	355	20180	383	15077	438
0.2 T	44864	339	28568	432	25593	395
0.3 T	53764	338	37693	397	33684	448
0.5 T	62247	316	44341	679	40437	629
0.7 T	71665	586	50764	590	48837	620

Applied Field	-90°		90°	
	DF	INT	DF	INT
0 T	1780	337	2041	352
0.1 T	1834	398	2507	434
0.2 T	1695	412	2187	466
0.3 T	1675	433	2345	451
0.5 T	1655	387	2303	396
0.7 T	1476	348	2252	390

V. Conclusion

In this paper, the angular dependence of the critical current density for two samples of SuperPower's SCS4050-AP coated conductor is measured and compared. The asymmetric $J_c(B, \theta)$ behaviour is input into numerical models using two methods: data fitting using an engineering formula, and a simpler and more direct method using a two-variable direct interpolation. It is found that two methods are both accurate, but the two-variable direct interpolation is significantly faster than the data fitting method using an engineering formula, in some cases up to over 100 times faster. The direct interpolation method is therefore recommended as the best method to include anisotropic $J_c(B, \theta)$ behaviour to model HTS coated conductors in finite element models to achieve accurate, effective and efficient results.

REFERENCES

[1] D. Hu *et al.*, "DC characterization and 3D modelling of a triangular, epoxy-impregnated high temperature superconducting coil," *Supercond. Sci. Technol*., vol. 28, no. 6, Apr. 2015, Art. ID. 065011.

[2] Z. Hong, A. M. Campbell, and T. A. Coombs, "Numerical solution of critical state in superconductivity by finite element software", *Supercond. Sci. Technol.*, vol. 19, no. 12, pp. 1246-1252, Dec. 2006.

[3] R. Brambilla, F. Grilli, and L. Martini, "Development of an edge-element model for AC loss computation of high temperature superconductors," *Supercond. Sci. Technol.*, vol. 20, no.1, pp. 16-24, Jan. 2007.

[4] K. Kajikawa, T. Hayashi, R. Yoshida, M. Iwakuma, and K. Funaki, "Numerical Evaluation of AC losses in HTS Wires With 2D FEM Formulated by Self Magnetic Field", *IEEE Trans. Appl. Supercond*., vol. 13, no.2, pp. 3630-3633, Jun. 2003.

[5] R. Pecher, M. D. McCulloch, S.J. Chapman, L. Progozhin, and C. M. Elliott, "3D-modelling of bulk type-II superconductors using unconstrained H-formulation," *Proc. 6th European Conf. on Applied Superconductivity*, pp. 1-11, Sep. 2003.

[6] M. D. Ainslie *et al*., "An improved FEM model for computing transport AC loss in coils made of RABiTS YBCO coated conductors for electric machines," *Supercond. Sci. Technol*., vol. 24, no. 4, Apr. 2011, Art. ID. 045005.