Extrapolative Scaling Expression:
A fitting equation for extrapolating full $I_c(B,T,\varepsilon)$ data matrixes from limited data

Jack Ekin, Najib Cheggour, Loren Goodrich, Jolene Splett
*NIST, Univ. Colorado - Boulder*

Bernardo Bordini, David Richter, and Luca Bottura
*CERN - Geneva*

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Organization of Talk

1. **Context for Extrapolative Scaling Expression (ESE, or “easy”)**
   Result of three SUST invited topical reviews:
   - Part 1 – Organization of many parameterizations of USL into separable parts
   - Part 2 – Derivation of ESE from raw scaling data (> 4000 \( I_c \) measurements)
   - Part 3 – Applications

2. **Focus on new extrapolation capabilities made possible with ESE**
   - Emphasis on *concatenation of errors* (not included in SUST articles)
   - Illustrate with practical conductors: HL-LHC, ITER, NMR cryo-cooled magnets

3. **Suggestions for future research**
Two approaches in use to analyze $I_c(B,T,\varepsilon)$

Pinning force curves $F_p(B,T,\varepsilon) = I_c(B,T,\varepsilon) B$

**Fundamental Scaling**

Curve registration

1) $F_{p_{\text{max}}}(T,\varepsilon)$  
   All we get

2) $B_{c2}(T,\varepsilon)$

Extrapolation capability

**Master scaling curve**

**Global Fitting Equations**

\[
I_c(B,T,\varepsilon) B = C [b_{c2}(\varepsilon_0)]^s \times 
\times (1 - t^n)^{\eta+m} (1 - t^2)^{\mu} b^p (1-b)^q
\]

**Interpolation only**

**No master scaling curve**

**Global Fitting Equations**

(a) Pinning force curves

(b) Curves scaled using raw scaling data for $B_{c2}(T,\varepsilon)$ and $F_{p_{\text{max}}}(T,\varepsilon)$

(c) Curves scaled using global-fit values for $B_{c2}(T,\varepsilon)$ and $F_{p_{\text{max}}}(T,\varepsilon)$
ESE is a fitting equation for the 3-dimensional $I_c(B,T,\varepsilon)$, which is:

1. Derived from an extensive one-time analysis of raw scaling data.

2. But simply applied as a fitting equation (without analyzing raw scaling data).

3. And, unlike present fitting equations, it has the **extrapolation capability** of fundamental scaling. (Reason? – based on master scaling curves; it is not empirical or semi-empirical)

Because no theoretical assumptions were made in its derivation, the results also serve to evaluate underlying semi-theoretical models for general par. of USL
Derivation of the Extrapolative Scaling Expression (ESE)

True Scaling

Registration gives:

1) \( K(T,\varepsilon) \)

2) \( B_{c2}^*(T,\varepsilon) \)

- Extrapolation capability

Three scaling constants:

\[ w = 3.0 \pm 0.03 \]
\[ v = 1.5 \pm 0.04 \]
\[ u = 1.7 \pm 0.1 \]

ESE Fitting Equation

\[ I_c(B, T, \varepsilon) B = C b_{c2}(\varepsilon)^s (1 - t^{1.5})^{\eta - \mu}(1 - t^2)^\mu b^0(1 - b)^q \]

Reduced variables:

\[ b \equiv B/B_{c2}(T, \varepsilon) \]
\[ t \equiv T/T_{c2}^*(\varepsilon) \]

Where:

\[ B_{c2}(T, \varepsilon)/B_{c2}(0, 0) = (1 - t^{1.5}) b_{c2}(\varepsilon) \]
\[ T_{c}^*(\varepsilon) = T_{c}^*(0) b_{c2}(\varepsilon)^{1/3} \]

Stable with respect to:

- conductor type
- trim factors
- \( p \) and \( q \) values
- magnetic self-field correction
Magnetic Self-field Correction
Needed for comparisons -- short-sample data, different apparatus, magnetization

Large effect on flux-pinning curve, BUT:

1. $F_P$ curves **still scale** into master curve
2. Scaling constants $w$, $v$, and $u$ unchanged by SF correction

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**Graphs**

**a** Not corrected

- Luvata ITER Temperature Dataset
- No magnetic self-field correction
- Master Scaling Curve
- Trim $F_0 < 125$ AT, $\rho = 0.56$ and $q = 1.72$
- Temperature data only
- $T = 2.26$ K to 14 K

**b** Corrected

- Luvata ITER Temperature Dataset
- Magnetic self-field corrected
- Master Scaling Curve
- Trim $F_0 < 125$ AT, $\rho = 0.43$ and $q = 1.53$
- Temperature data only
- $T = 2.26$ K to 14 K
Bottom line: raw scaling analysis gives:

**Extrapolative Scaling Expression (ESE), the “easy” fit.**

Most useful form:

\[ I_c(B,T,\varepsilon) B = C b_{c2}(\varepsilon)^s (1 - t^{1.5})^{\eta - 1} (1 - t^2) b^p (1 - b)^q \]

where \( b = B/B_{c2}^*(T,\varepsilon) \) is the reduced field, and \( t = T/T_{c}^*(\varepsilon) \) is the reduced temperature

\[ B_{c2}^*(T,\varepsilon)/B_{c2}^*(0,0) = (1 - t^{1.5}) b_{c2}(\varepsilon) \]

\[ T_{c}^*(\varepsilon) = T_{c}^*(0) b_{c2}(\varepsilon)^{1/3} \]

and fitting parameters \( C \) & \( B_{c2}^*(0,0) \), and 4 core parameters \( T_{c}^*(0) \), \( s \), \( \eta \), & \( C_1 \) (in \( b_{c2}(\varepsilon) \)).
**Hybrid temperature models** with \( \eta \) fitted (Durham) and \( \mu = 1 \) (Twente) have the following advantages:

- Overall fitting accuracy
- Parameter consistency (\( \eta \) variability < half that of \( \mu \))
- **Extrapolation** capability to temperatures below 4 K (~1 % errors)

**Exponential strain model** for \( b_{c2}(\varepsilon) \):

- One fitting parameter \( C_1 \) (*strain sensitivity index, default values*)
- 3-D strain capability
- **Extrapolation** capability to high compressive strains
Applications of the Extrapolative Scaling Expression (ESE)

N.B. -- Fitting $F_p$, not $I_c$. Errors consistently one-fifth!

*Extrapolation* capability in four new areas:

1. Five-fold reduction in measurement space: extrapolate minimum dataset
   (reduces weeks for full $I_c(B-T-\epsilon)$ measurements to a few days)
2. Combination of data from separate $T$ and $\epsilon$ apparatuses
   (offers flexibility and productive use of limited data)
3. Full $I_c(B,T,\epsilon)$ extrapolation from as little as a single $I_c(B)$ curve
   (useful for production sample measurements, e.g., HL-LHC, FCC)
4. Interpolation with option for nearby extrapolations
   (with default core parameters)
Minimum Dataset for extrapolating full $I_c(B,T,\varepsilon)$ characteristics – derived from scaling
Visualize with $T-\varepsilon$ measurement map
5-fold reduction in measurement space
Minimum dataset extrapolation with ESE-Hybrid model

RMSE $F_p = 0.166\%$
RMSE $I_{c12T} = \sim 3\text{A}$

WST-ITER
$\varepsilon = -0.445\%$

Critical Current (A)

Self-field corrected $B$ (T)

Temperature (K)

- 1.90
- 4.03
- 6.00
- 8.00
- 10.00
- 12.00
Applications of the Extrapolative Scaling Expression (ESE)

*Extrapolation* capability in three new areas:

1. Five-fold reduction in measurement space for unified *B*-*T*-*ε* apparatuses
   (reduces weeks for full $I_c(B-T-ε)$ measurements to a few days)

2. Combination of data from separate *T* and *ε* apparatuses
   (offers flexibility and productive use of limited data)

3. Full $I_c(B,T,ε)$ extrapolation from as little as a single $I_c(B)$ curve
   (useful for production measurements, e.g., HL-LHC, FCC)

4. Interpolation with option for nearby extrapolations
   with default *core* parameters
Combining limited datasets (examples in Part 3)

**Core parameters** – depend only on ratios of raw scaling data. **Very stable.**

Transfer among similar conductors (same comp., config, and heat treatment)

<table>
<thead>
<tr>
<th>Available data</th>
<th>Parameters determined</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$I_c(B,T,\varepsilon)$ (unified $T,\varepsilon$ apparatus)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c(B,T)$ fixed $\varepsilon$ (dedicated $T$ rig)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c(B,\varepsilon)$ fixed $T$ (dedicated $\varepsilon$ rig)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c(T)$ fixed $B,\varepsilon$ (dedicated $T$ rig)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c(\varepsilon)$ fixed $B,T$ (dedicated $\varepsilon$ rig)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c(B)$ fixed $T,\varepsilon$ (routine $I_c$ testing)</td>
<td>✓</td>
</tr>
<tr>
<td>$I_c$ fixed $B,T,\varepsilon$ (routine $I_c$ testing)</td>
<td>✓</td>
</tr>
</tbody>
</table>

Min. dataset: Single $I_c(B)$ curve
Single $I_c(B)$ curve extrapolation

Each point is a complete plot of $I_c - B$ (and $F_p - B$)

OST-RRP®

Zero applied strain

Liquid He temp.

Try to predict

Single $I_c(B)$ curve!
Single $I_c(B)$ curve extrapolation
From point in $T$-$\varepsilon$ map at 4.07 K and 0.035 % strain
Core parameters from minimum dataset

Critical Current (A)

Self-field corrected $B$ (T)

OST-RRP
$\varepsilon = 0.035\%$

Critical Current (A)

$RMSE = 0.117\%$
$RMSE I_{c12T} = \sim 5A$

Invited presentation 5MOr1A-01 given at ASC 2016; Denver, Colorado, USA, September 4 – 9, 2016.
Each point is a complete plot of $I_c - B$ (and $F_p - B$)

Single $I_c(B)$ curve extrapolation
Core parameters from min. dataset with data only > 4 K

Zero applied strain

Try to predict
Single $I_c(B)$ curve extrapolation from 4.07 K and 0.035% strain

Core par. from minimum dataset, using data only $> 4$ K (combine 3 extrapolations)

$\varepsilon = 0.035\%$

$OST-RRP$

$RMSE = 0.122\%$

$RMSE I_{c12T} = \sim 6 A$
Single $I_c(B)$ curve strain extrapolation
Core par. from minimum dataset, using data only > 4 K

Each point is a complete plot of $I_c - B$ (and $F_p - B$)

Zero applied strain

Liquid He temp.

Single $I_c(B)$ curve

Try to predict at 2.45 K
Single $I_c(B)$ curve strain extrapolation from 4.07 K and 0.035 % strain

Core parameters from minimum dataset, using data only > 4 K

RMSE = 0.122%
RMSE $I_{c12T}$ = ~6 A
Caveats:

1. Evaluated intrinsic errors for predicting the non-core parameters from a single $I_c(B)$ curve

2. Extrinsic errors need to be minimized. Core parameters determined from:
   - Samples with similar configuration, doping, and heat treatment (e.g., production samples).
   - Similar sample holders (minimize strain variability)
     Matching material preferred (thermal contraction strain)
     Continuously soldered preferred to provide good $F_L$ support
     Cu-Be holders easy solution (avoids unsupported conductor settling)

→ If control extrinsic errors, such extrapolations quite effective for similar conductors.
Applications of the Extrapolative Scaling Expression (ESE)

*Extrapolation* capability in three new areas:

1. Five-fold reduction in measurement space for unified $B$-$T$-$\epsilon$ apparatuses
   (reduces weeks for full $I_c(B$-$T$-$\epsilon$) measurements to a few days)

2. Combination of data from separate $T$ and $\epsilon$ apparatuses
   (offers flexibility and productive use of limited data)

3. Full $I_c(B,T,\epsilon$) extrapolation from as little as a single $I_c(B)$ curve
   (useful for production measurements, e.g., HL-LHC, FCC)

4. Interpolation with option for nearby extrapolations
   with default core parameters when data limited
Table A1.1. The ESE parameter set, with Hybrid $k(t)$ and the **Exponential** parameterization of $b_{c2}(e)$ for data not corrected for magnetic selffield.

<table>
<thead>
<tr>
<th>Nb$_3$Sn Conductor</th>
<th>$C$ (AT)</th>
<th>$B_{c2}^*(0,0)$ (T)</th>
<th>$T_c^*(0)$ (K)</th>
<th>$\eta$</th>
<th>$s$</th>
<th>$\epsilon_{m^{**}}$ (%)</th>
<th>$C_1^2$</th>
<th>$p^\dagger$</th>
<th>$q^\dagger$</th>
<th>RMSFD (%)</th>
<th>RMS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OST-RRP©</td>
<td>50,514</td>
<td>29.09</td>
<td>16.94</td>
<td>2.254</td>
<td>1.150</td>
<td>-0.355</td>
<td>0.748</td>
<td>0.5</td>
<td>2.061</td>
<td>9.0</td>
<td>0.120</td>
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<tr>
<td>WST-ITER</td>
<td>21,015</td>
<td>31.02</td>
<td>16.81</td>
<td>2.025</td>
<td>1.388</td>
<td>-0.302</td>
<td>0.817</td>
<td>0.573</td>
<td>1.834</td>
<td>4.8</td>
<td>0.114</td>
</tr>
<tr>
<td>LUVATA</td>
<td>14,955</td>
<td>29.70</td>
<td>16.43</td>
<td>1.966</td>
<td>1.4</td>
<td>-0.321</td>
<td>0.657</td>
<td>0.562</td>
<td>1.703</td>
<td>2.0</td>
<td>0.078</td>
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<tr>
<td>VAC</td>
<td>7,631</td>
<td>29.91</td>
<td>16.84</td>
<td>2.002</td>
<td>1.097</td>
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<td>0.480</td>
<td>1.445</td>
<td>4.6</td>
<td>0.247</td>
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<tr>
<td>EMLMI</td>
<td>11,920</td>
<td>30.79</td>
<td>17.02</td>
<td>2.380</td>
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<td>0.5</td>
<td>1.835</td>
<td>3.6</td>
<td>0.170</td>
</tr>
</tbody>
</table>

Table A1.2. The ESE parameter set, with Hybrid $k(t)$ and the **Invariant** parameterization of $b_{c2}(e)$ for data not corrected for magnetic selffield.

<table>
<thead>
<tr>
<th>Nb$_3$Sn Conductor</th>
<th>$C$ (AT)</th>
<th>$B_{c2}^*(0,0)$ (T)</th>
<th>$T_c^*(0)$ (K)</th>
<th>$\eta$</th>
<th>$s$</th>
<th>$\epsilon_{m^{**}}$ (%)</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$p^\dagger$</th>
<th>$q^\dagger$</th>
<th>RMSFD (%)</th>
<th>RMS (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OST-RRP©</td>
<td>47,954</td>
<td>27.58</td>
<td>16.65</td>
<td>2.252</td>
<td>1.210</td>
<td>0.302</td>
<td>1.016</td>
<td>0.717</td>
<td>0.183</td>
<td>0.5</td>
<td>2.061</td>
<td>7.3</td>
<td>0.104</td>
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<tr>
<td>WST-ITER</td>
<td>19,772</td>
<td>29.62</td>
<td>16.53</td>
<td>2.023</td>
<td>1.356</td>
<td>0.305</td>
<td>0.823</td>
<td>0.424</td>
<td>0.118</td>
<td>0.577</td>
<td>1.855</td>
<td>4.5</td>
<td>0.106</td>
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<tr>
<td>LUVATA</td>
<td>14,166</td>
<td>28.60</td>
<td>16.21</td>
<td>1.966</td>
<td>1.4</td>
<td>0.323</td>
<td>0.660</td>
<td>0.669</td>
<td>1.136</td>
<td>0.562</td>
<td>1.709</td>
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<td>0.082</td>
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<tr>
<td>VAC</td>
<td>7,654</td>
<td>28.92</td>
<td>16.45</td>
<td>1.972</td>
<td>1.040</td>
<td>0.311</td>
<td>0.893</td>
<td>0.376</td>
<td>0.053</td>
<td>0.512</td>
<td>1.549</td>
<td>4.5</td>
<td>0.219</td>
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<tr>
<td>EMLMI</td>
<td>11,419</td>
<td>28.81</td>
<td>16.71</td>
<td>2.405</td>
<td>0.851</td>
<td>0.273</td>
<td>1.051</td>
<td>0.610</td>
<td>0.258</td>
<td>0.5</td>
<td>1.883</td>
<td>3.6</td>
<td>0.156</td>
</tr>
</tbody>
</table>
Default Core Parameters

Survey of core values for fully optimized ternary high-$J_c$ Nb$_3$Sn $\rightarrow$ average default default values:

$$T_c^*(0) = 16.7 \text{ K}$$
$$\eta = 2.0 \text{ (ITER)} - 2.2 \text{ (RRP)}$$
$$s = 1.2 \text{ (RRP)} - 1.4 \text{ (ITER)}$$
$$p = 0.5 \text{ and } q = 2.0.$$  

Additional meas. $\rightarrow$ “catalog” by generic conductor category  
(e.g., Ti vs. Ta doping, RRP, internal Sn, etc.)
Future Work

Immediate need: (huge dividends)
1. * Measure $I_c(B,T)$ above 4.2 K for at least one conductor of the RRP and PIT production wires for the Hi-Lumi (to obtain $T_c^*$ and $\eta$).

Longer term:
2. Compile core parameters in different types of Nb$_3$Sn – catalog values
3. Evaluate accuracy of ESE in extreme regions of $B$-$T$-$\epsilon$ space for magnet modeling
4. Magnetization vs. transport $I_c$ data
5. Assess if scaling constants hold for artificial-pinning-center architectures
6. ESE relationship for BSCCO, MgB$_2$, Nb$_3$Al, YBCO? (master curve $\rightarrow$ extrapolation)

Conclusion
• ESE is based on fundamental raw scaling data
• But unlike fundamental scaling, applied as a fitting equation– quick, straightforward
• Simple, robust, and
• Can interpolate and extrapolate with excellent accuracy $\rightarrow$ significant time savings

Excel source data & ESE spreadsheet tool at www.ResearchMeasurements.com
SUST invited topical review articles