Power dissipated by trapped vortices under a strong RF field and Campbell penetration depth in superconducting resonant cavities

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Campbell’s penetration depth: good work always stays young and popular
Superconducting linac applications

Spallation neutron source (ORNL)

X-ray free electron laser

Superconducting LINAC

Tunable 0.25-14μm light source at JLab
Superconducting RF cavities resonating at 0.1-2 GHz
Currently made of pure niobium
Cooled by superfluid helium at 2K
Tens of thousands of these in miles long tunnels
**Definitions**

Δ – superconducting gap

ξ – coherence length

λ – magnetic penetration depth

\( \rho_n \) - normal state resistivity

\( R_s \) - surface resistance

\( Q \) – quality factor

\( \omega = 2\pi f \) – RF circular frequency

\( \eta_0 = \phi_0^2 / 2\pi \xi^2 \rho_n \) - Bardeen-Stephen vortex drag coefficient

\( \phi_0 \) - magnetic flux quantum

\( \ell \) - spacing between a pinning center and the surface

\( \nu_0 \) - Larkin-Ovchinnikov (LO) critical velocity of a vortex

\( \kappa \) – thermal conductivity

\( d \) – thickness of a cavity wall

\( \alpha_K \) - Kapitza thermal conductance between a cavity wall and liquid He

\( \epsilon = g \phi_0^2 / 4\pi \mu_0 \lambda^2 \) - vortex line tension

\( g = \ln(\lambda/\xi) + 1/2 \)

\( T_c \) - critical temperature

\( B_{c1} \) - lower critical field

\( B_c \) - thermodynamic critical field

\( B_s \) - superheating field

\( B_{c2} \) - upper critical field
Quality factor

\[ Q = \frac{\omega \mu_0 \int_V |\mathbf{H}(\mathbf{r})|^2 dV}{\int_A R_s |\mathbf{H}(\mathbf{r})|^2 dA} = \frac{G}{\langle R_s \rangle}, \]

Mean EM energy

Mean dissipated power

Vacuum impedance

Surface resistance of good normal metals

\[ R_s = (\pi \mu_0 f \rho_n)^{1/2} \]

Clean Cu with \( \rho_n = 10^{-10} \) Ωm at \( f = 0.5-2 \) GHz has \( R_s = 0.5-1 \) mΩ

Exponentially-small BCS surface resistance of superconductors:

\[ R_s \sim \frac{\mu_0^2 \omega^2 \lambda^3}{\rho_n kT} \ln \left( \frac{9kT}{2\hbar\omega} \right) \exp \left( -\frac{\Delta}{kT} \right) \sim 2 - 10 n\Omega, \quad @ 1.7 - 2K, 1 - 2GHz \]

\[ Q \sim 10^{10} - 10^{11} \]
How good can Nb cavities be?

Continuous progress in improving $Q(H)$ and $E_{\text{acc}}$ in Nb cavities.

Understanding the fundamental limits of $Q(H)$ and the SRF accelerating gradients

The RF field of $H = 200$ mT induces current densities at the surface close to the BCS pairbreaking limit.

High $Q$ can only be achieved in the Meissner state with a small density of trapped vortices.
Why are trapped vortices so bad for SRF cavities?

- Vortices get trapped by materials defects on cooling the cavity through $T_c$ at which $H_{c1}(T)$ vanishes.

- Trapped vortices caused by Earth’s magnetic field can produce $\sim 10^2$ higher RF losses than the BCS surface resistance at 2K and 1-2 GHz.

- Even good screening (1% of $H_E$) cannot eliminate trapped vortices. Temperature maps have revealed sparse hotspots of vortex bundles which reduce the quality factors and breakdown fields: Vogt, Kugeler and Knobloch, PRAB 18, 042001 (2015); Gonnella, Kaufman and Liepe, JAP 119, 073904 (2016); Dhakal et al, PRAB 23, 023102 (2020).
Detection and manipulation of trapped vortices

In films vortices are observed using scanning SQUID, *Kirtley, Rep. Prog. Phys. 73, 126501 (2010)*
MO imaging, STM, MF, Lorentz microscopy, ...

Arrays of carbon sensors to get local temperature maps with the sensitivity of a few mK and spatial resolution of a few mm (Cornell, Jlab, FNAL)

Key issues

- Trapped vortices can produce significant losses which can be much higher than the BCS losses in SRF resonator cavities.

- Vortex losses are determined by an effective Campbell penetration depth

- New physics of superfast vortices driven by strong Meissner screening currents at the depairing limit in SRF cavities.

- How fast can vortices move? How long does it take for a vortex to penetrate a superconductor?

- Nonlinear dynamics of supersonic vortices: field-dependent RF losses, Larkin-Ovchinnikov instability, decrease of the surface resistance with the RF amplitude, ...

- How much vortex dissipation can be tolerated? Can vortex dissipation be mitigated by strong pinning?
Trapped vortex driven by RF Meissner current

An elastic vortex is driven by the Lorentz force \( \mathbf{f}_L = \phi_0 \mathbf{J} \times \mathbf{z} \) perpendicular to \( \mathbf{J} \):

\[
J(z, t) = \left( \frac{H_a}{\lambda} \right) e^{-z/\lambda} \sin \omega t
\]

The surface Lorentz force is balanced by viscous drag force and bending stress.

At \( H_a = 100-200 \text{ mT} \), \( J(0) \) approaches the depinning limit:

\[
J_d \approx \frac{H_c}{\lambda}
\]

Typical depinning \( J_c = 10-100 \text{ kA/cm}^2 \) in Nb are some 4 orders of magnitude lower than \( J_d = H_c/\lambda, = 500 \text{ MA/cm}^2 \).

Pinning is too weak to stop the vortex tip at the surface above \( H > 0.01H_c = 2 \text{ mT} \).
RF Campbell length

Dynamic eq for displacements $u(x,t)$ of a vortex driven by a weak RF field $H_a \ll H_c$

$$\eta \ddot{u} = \epsilon u'' - \frac{H_a}{\lambda} e^{-x/\lambda} \sin \omega t$$

Elastic RF ripple length – Campbell penetration depth:

$$L_\omega = \sqrt{\frac{\epsilon}{\eta \omega}} \approx \frac{\xi}{2\lambda} \sqrt{\frac{g\rho_n}{\pi \mu_0 f}}$$

- **Clean Nb**

  $$\lambda \approx \xi, \quad \rho_n = 1 \text{ n}\Omega m, \quad f = 2 \text{ GHz}$$

  $$L_\omega \approx 180 \text{ nm}$$

- **Nb$_3$Sn**

  $$\frac{\lambda}{\xi} \approx 20, \quad \rho_n = 0.2 \mu\Omega m, \quad f = 2 \text{ GHz}$$

  $$L_\omega \approx 126 \text{ nm}$$

- Campbell length $L_\omega$ can be much greater than $\lambda$.

- $L_\omega$ can be either larger or smaller than the pin distance from the surface. If $\ell > L_\omega$, the effect of pinning is weak.
Low-field RF power of an oscillating vortex

- Low frequencies. The whole vortex segment swings:
  \[ P \approx \frac{4\pi B_p^2 \ell^3 \omega^2}{3 \rho_n \xi^2} \]

  Decreases strongly as the pin spacing decreases

- Intermediate \( \omega \): \( \lambda \lesssim L_\omega \lesssim \ell \)
  \[ P \approx \pi \mu_0^{-3/2} B_p^2 \lambda \xi \sqrt{\omega \rho_n} \]

  No dependence on the pin spacing

- High \( \omega \): \( L_\omega \lesssim \lambda \)
  \[ P_\infty = \pi H^2 \rho_n \xi^2 / 2 \lambda \]

  No dependence on the pin spacing

\[ P \sim 0.13 \, \mu W \text{ at } B = 100 \, \text{mT and 2 GHz.} \]

Hotspots revealed by thermal maps require regions \( \sim \) few mm with \( \sim \) 10^6 vortices.

Gurevich and Ciovati. PRB 87, 054502; (2013)
Extreme dynamics of vortex tips at the surface

At $H = H_c$, the superflow velocity of Cooper pairs reaches the critical pairbreaking value $v_c = \Delta/p_F$.

How fast can the vortex tip move at the pairbreaking limit?

$$v \sim \frac{J_d \phi_0}{\eta} \sim \frac{\rho_n \xi}{2\mu_0 \lambda^2}$$

This rough estimate yields $v = 10 \text{ km/s}$, which exceeds both the speed of sound ($2$-$4 \text{ km/s}$) and $v_c = \Delta/p_F = 1 \text{ km/s}$.

How can a supersonic vortex tip remain connected to a subsonic elastic vortex line in the bulk?

SRF cavities are a unique testbed to study the extreme dynamics of a vortex driven by non-dissipative Meissner currents at the pairbreaking limit.
How can a vortex move faster than the current superflow which propels it?

- Vortex core stretches along the direction of motion
- Vortex can move much faster than the drift velocity of supercurrent
- V can exceed the pairbreaking velocity

A sailboat can move much faster than the wind if drag is weak and the sail is nearly perpendicular to the wind blow.
What does experiment say?

75 nm thick Pb film: imaging of penetrating vortices with a nanoscale SQUID on tip

Velocities can reach $10-20\text{ km/s}$ as $J(x,y)$ at the edge reaches $J_d$ ($H = H_s$ for the SRF cavities)

If $v = 10\text{ km/s}$, a vortex penetrates by the distance

$$L \simeq \frac{v}{f} \simeq 10\mu m \gg \lambda, \quad @ 1GHz$$

Vortices penetrate almost instantaneously through the Meissner RF layer

Hot vortex branching trees. No materials defects can stop such superfast vortices.
Dynamics of vortex branching observed by SOT microscope

Pb bridge at $B_a = 27 \text{ G}$
SOT diameter: $225 \text{ nm}$
Scan area: $12 \times 12 \mu\text{m}^2$
Pixel size: $40\text{ nm}$
Scan time: $4 \text{ min/frame}$
$T = 4.2 \text{ K}$

What happens to the vortex core at high velocities?

- A nearly round vortex core of radius $\approx \xi$
- A cloud of dissipative quasiparticles is locked onto the moving core
- The core stretches along $\mathbf{v}$ as the recovery length of $\Delta(x, t)$ behind the core increases with $v$: $L_\Delta \simeq v\tau_\Delta$, $v > \xi/\tau_\Delta$
- A cloud of diffusive nonequilibrium quasiparticles is lagging behind the core
- Vortex drag decreases with $v$: $\eta(v) \simeq \frac{\phi_0^2}{2\pi\rho_n\xi L_\Delta}$
Velocity dependence of $\eta(v)$

Larkin-Ovchinnikov mechanism

Reduction of the vortex drag due to diffusive depletion of quasiparticles in the moving core

$$\eta(v) \simeq \frac{\eta_0}{1 + (v/v_0)^2}$$

LO critical velocity:

$$v_0 \sim (D/\tau_E)^{1/2}(1 - T/T_c)^{1/4}$$

D is the electron diffusivity

The energy relaxation time $\tau_E(T)$ caused by inelastic e-p scattering increases as $T$ decreases so $v_0(T)$ is expected to decrease at $T \ll T_c$

Larkin and Ovchinnikov. JETP 41, 960 (1975)

Electron overheating

The drag coefficient depends on the electron temperature $T_v$ of the vortex

$$\eta(T_0) = \frac{\phi_0 B_{c2}(0)}{\rho_n} \left( 1 - \frac{T_v}{T_c} \right)$$

2D power balance:

$$\eta(T_v) v^2 \simeq (T_v - T_0) \kappa$$

An effective thermal conductivity $\kappa(T_0)$ contains both quasiparticle and phonon contributions

Solving for $T_v$ yields $\eta(v)$ in the LO form with

$$v_0 = \sqrt{T_c \kappa(T_0)/\eta_0(0)}$$

Kunchur, PRL 89, 137005 (2002)
Gurevich and Ciocci, PRB 77, 104501 (2008)
Larkin-Ovchinnikov instability

Balance of drag and Lorentz forces for a straight vortex in a thin film:

\[
\frac{\eta_0 v}{1 + \left(\frac{v}{v_0}\right)^2} = \phi_0 J
\]

Observations on different materials:
Musienko et al., JETP Lett. 31, 567 (1980); Klein et al., JLTP 61, 413 (1985);
Amenio et al., PRB 76, 054502 (2007); Grimaldi et al., J Phys C97, 012111 (2008);
Villard et al., JLTP 131, 957 (2003); Doettinger et al., PRL 76, 1691 (1994);

\[ J_m = \frac{\eta_0 v_0}{2\phi_0} \]

Acceleration of a runaway vortex at \( v > v_0 \), jumps on the V-I curves

The observed \( v_0(T) \) is \( \approx 0.1 - 1 \text{km/s} \) near \( T_c \) and decreases as \( T \) decreases.

Can be masked by heating effects

dc transport on Nb films:
Peroz and Villard, PRB 72, 014515 (2005)
LO instability of a trapped vortex

Since the LO critical velocity $v_0 \sim 0.1 - 1$ km/s is 1-2 orders of magnitude smaller than velocities of a vortex at $H = 10 - 100$ mT, the LO instability can be essential in SRF cavities.

- What happens to the vortex if its fast tip is LO-unstable while the rest of the vortex is LO-stable?
- Can a vortex be shredded into disconnected pieces by strong surface current?
- Dependence of RF losses and the residual surface resistance caused by trapped vortices on the RF field.
- The extreme vortex dynamics in SRF cavities is not masked by strong overheating typical of dc transport measurements at $T \ll T_c$. 
Nonlinear dynamic equations for a vortex

Balance of local forces perpendicular to a curvilinear vortex

\[ M \dot{v} + \eta(v)v = \epsilon/R - (H_a/\lambda)e^{-x/\lambda} \sin \omega t \]

Dynamic eq. for a dimensionless vertical displacement

\[ u(x,t) = y(x,t)/\lambda, \quad x \to x/\lambda : \]

\[ \mu \frac{\partial}{\partial t} \left( \frac{\dot{u}}{\sqrt{1 + u'^2}} \right) + \frac{\gamma \dot{u}\sqrt{1 + u'^2}}{1 + u'^2 + \alpha \gamma^2 \dot{u}^2} = \frac{u''}{(1 + u'^2)^{3/2}} - \beta e^{-x} \sin(2\pi t) \]

Takes into account vortex inertia, and nonlinearities of the LO vortex drag and bending rigidity

\[ \gamma = f/f_0, \quad f_0 = H_{c1}\rho_n/H_{c2}\lambda^2\mu_0 \]

\[ \alpha = (\lambda f_0/v_0)^2, \quad \beta = H_a/H_{c1} \]

\[ f_0 = 22 \text{ GHz for Nb.} \]

Pathirana and Gurevich, PRB 101, 064504 (2020)
Nonlinear vortex losses and residual resistance

Dissipated power per vortex:

\[ p = \int \langle \eta(v)v^2 \rangle ds \]

Surface resistance \( R_i \) for the mean trapped flux density \( B_0 \) is obtained from

\[ pB_0/\phi_0 = R_i H_a^2/2 \]

\[ R_i(\beta) = \frac{R_0 \gamma^2}{\beta^2} \int_0^1 dt \int_0^l \frac{(1 + u'^2)^{1/2} \dot{u}^2 dx}{1 + u'^2 + \alpha \gamma^2 \dot{u}^2}, \quad R_0 = \frac{2 \rho_n B_0}{\lambda B_{c2}} \]

For Nb at 1-2 GHz, we have \( \gamma \sim 10^{-1}, \) and \( \alpha \sim 10^2 - 10^4. \) At small \( f \) and \( H_a \) the LO term in the denominator is negligible and \( R_i \) is independent of \( H_a \)

As \( H_a \) and \( f \) increase, \( \dot{u}^2 \) cancels out and \( R_i \) becomes nearly independent of frequency and decreases with the RF field:

\[ R_i \propto H_a^{-2} \]
LO mechanism of the low-field Q(H) rise

The surface resistance $R_i(H)$ starts decreasing with the field amplitude as the frequency increases. Calculated for different values of

$$\gamma = \frac{f}{f_0} \ @ \ l = 4\lambda, \ \alpha = 3 \cdot 10^3$$

Fit to the experimental data of for a 1.47 GHz Nb cavity

Ciovati, JAP 96, 1591 (2004)

$$l = 3\lambda, \ B_0 = 0.73 \mu T,$$

$$v_0(2K) = 30 \text{ m/s},$$

$$v_0(1.37K) = 35 \text{ m/s},$$
Effect of frequency on the field dependence of $R_i(H_a)$

Transition from quasi-harmonic to relaxation oscillations at the peak in $R_i(H)$. The Campbell length increases with $H_a$:

$$L_\omega(H_a) \simeq \sqrt{\epsilon/\omega\eta(v)}$$

- $L_\omega(H_a, \omega) < l$ before the peak
- $L_\omega(H_a, \omega) > l$ after the peak
Tuning the LO vortex dynamics by impurities

\[ \gamma_0 = 0.004 \]

Calculated for \[ \alpha_0 = 1.6 \cdot 10^4, \quad l = 3\lambda_0 \]

- Making the surface dirtier and decreasing the impurity mean free path shifts the anomalous drop of the vortex surface resistance with \( H_a \) to lower fields.
- May pertain to the low-field Q(H) drop observed on many Nb cavities.
Effect of weak overheating on the surface resistance

Thermal feedback for trapped vortices and BCS:

\[
\left( R_i(H_a) + R_0 e^{(T - T_0) \Delta / T_0^2} \right) \frac{H_a^2}{2} = (T - T_0) g,
\]

vortices  \hspace{1cm} \text{BCS}

Thermal resistance of the cavity wall:

\[
g^{-1} = \alpha_K^{-1} + \frac{d}{\kappa}
\]

- Interplay of the descending \( R_i(H) \) and ascending \( R_0(H_a) \) due to overheating produces a minimum in \( R_s(H_a) \)
- At large LO critical velocity \( v_0(T) \), BCS overheating reverses the decrease of \( R_s(H_a) \) with \( H_a \)
- Too many trapped vortices cause strong overheating which can eliminate the minimum in \( R_s(H) \) as \( v_0(T) \) increases with \( T \)
Could strong pinning be effective in SRF cavities?

**GOOD**

- Artificial pinning centers (APCs) which take 10% of current-carrying cross-section can produce critical current densities $J_s \approx 0.1 J_d$

- For cavities this can only be effective below the depinning field $H < 0.1 H_c = 20 \text{ mT} = 10\%$ of the SRF breakdown field for Nb.

- Reduction of vortex losses only in a small low-H part of the field operation range

**BAD**

- 10% of metallic APCs produce huge ohmic losses above the proximity effect breakdown field. Incompatible with high Q controlled by the BCS surface resistance

- 10% of dielectric APCs block the current-carrying cross section, greatly increasing the field penetration depth and the BCS surface resistance

- Above the depinning field, high Bean's hysteretic losses make high-$J_c$ SRF cavities no better than the normal Cu cavities

$\alpha$-Ti ribbons in a Nb-Ti alloy *(D. Larbalestier & P. Lee)*
Conclusions

- At RF fields of 100-200 mT tips of vortices trapped in Nb cavities can reach velocities of a few km/s approaching the speed of sound (3.5 km/s)

- Extreme nonlinear dynamics of the elastic vortex, drastic change of a hot moving vortex core, strong pairbeaking effects and nonequilibrium kinetics of quasiparticles.

- Decrease of the residual surface resistance due to the Larkin-Ovchinnikov mechanism and electron overheating in the vortex core.

- The descending field dependence of the surface Ri(H) develops as the frequency increases. A new mechanism of the Q(H) rise which can be tuned by impurities.

- Pinning at the surface can only reduce vortex dissipation at low RF fields << Hₜ.

- High-Q SRF cavities offer a unique opportunity to investigate the extreme dynamics of vortices at low temperatures.