Voltage Biased Superconducting Quantum Interference Device Bootstrap Circuit

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***Abstract* - We present a new dc Superconducting QUantum Interference Device (SQUID) readout circuit operating in the voltage bias mode and called SQUID Bootstrap Circuit (SBC). The SBC consists of two parallel branches: the first one consists of a dc SQUID with an inductively coupled feedback coil; it is used for the enhancement of the flux-to-current coefficient. The second branch consisting of a shunt resistor and a coil inductively coupled to SQUID suppresses the preamplifier noise current by increasing the dynamic resistance. Consequently, the SBC effectively reduces the preamplifier noise below the SQUID intrinsic noise. For helium-cooled planar SQUID magnetometer with a SQUID inductance of 350 pH, flux noise of about 4 $\mu\Phi_0/\sqrt{Hz}$ and magnetic field resolution of 3 fT/√Hz, were obtained. The SBC leads to a simple and convenient readout electronics for dc SQUID.**

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I. INTRODUCTION

The main challenge faced by the dc Superconducting QUantum Interference Device (SQUID) readout electronics is to amplify the small SQUID output signals without introducing additional noise from the room-temperature preamplifier. That noise contribution must be kept below the intrinsic noise of the SQUID. A dc SQUID can be operated in two modes, the current bias mode, and the voltage bias mode. With regard to the intrinsic noise of the SQUID, there is, in principle, no difference between the two modes [1]. In order to reduce the preamplifier noise, a transformer to match the low impedance of the SQUID to the high impedance of the preamplifier is used in the so-called flux modulation scheme [2]. Some other approaches are described in detail in [1], *e.g.*, the two-stage configuration [3], the double relaxation oscillation SQUID [4], and further readout schemes.

The direct readout with additional positive feedback (APF) in the current bias mode [5], and the noise cancellation scheme (NC) in the voltage bias mode [6], simplify the readout electronics, because flux modulation is no longer needed. The APF increases the flux-to-voltage coefficient of the SQUID at the working point of an asymmetric transfer function. This improves the impedance matching between the SQUID and the preamplifier. NC uses the same input circuit to suppress the noise contribution from the preamplifier. However, the principles and performance of APF and NC are quite different.

A dc SQUID can be considered as a flux-to-voltage ($\partial V/\partial \Phi$) converter and a nonlinear resistor R_d (the dynamic resistance of the SQUID), which are connected in series. If a constant bias voltage V_b is applied across the dc SQUID, an external magnetic flux ϕ_e causes a current change through the SQUID determined by $(\partial i/\partial \Phi) = (\partial V/\partial \Phi)/R_d$. The current i is periodically modulated by the external magnetic flux ϕ_e. Periodicity of the I-Φ characteristic is the flux quantum $\Phi_0 = 2.07 \times 10^{-15}$ Wb. Actually, in the voltage bias mode the SQUID acts as a flux-to-current ($\partial i/\partial \Phi$) converter.

The noise from the preamplifier contains two components, the voltage noise V_n and the current noise I_n*. Usually, V_n dominates the noise contribution of the preamplifier in the direct readout schemes. In the voltage bias mode, the equivalent flux noise of a preamplifier is expressed as $\delta\Phi_{\text{preamp.}} = V_n/[R_d \cdot (\partial i/\partial \Phi)]$. In this paper, we introduce a new circuit called the SQUID Bootstrap Circuit (SBC) to enhance the product of R_d and $\partial i/\partial \Phi$, and thus reduce the preamplifier noise. The main point of this work is the circuit analysis and principle demonstration. Our first experimental results are reported.

II. SQUID BOOTSTRAP CIRCUIT (SBC)

The SQUID Bootstrap Circuit consists of two parallel branches B_1 and B_2 shown in Figure 1(a): B_1 comprises the dc SQUID and the feedback coil of inductance L_1 connected in series; L_1 is inductively coupled to the SQUID loop with a mutual inductance M_1. Branch B_1 is shunted by B_2 consisting of a shunt resistor R_s and a coil of inductance L_2, which is coupled to the SQUID with a mutual inductance M_2. The room-temperature readout electronics is also shown in Figure 1(a) while the flux-locked loop (FLL) is omitted. The bias voltage source V_b is connected to the non-inverting input terminal of the preamplifier. The preamplifier acts as a current-to-voltage converter, and its conversion gain depends on R_g. The current *versus* flux characteristic of the SBC can be monitored from the preamplifier output V_m.

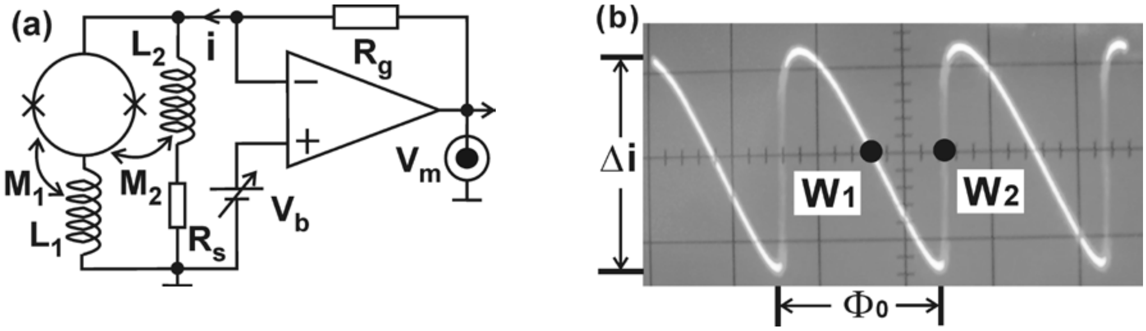

Fig. 1. (a) Voltage biased SBC circuit with preamplifier. (b) SBC I-Φ characteristic of B_1, measured at V_m under the critical condition of $M_1 \cdot (\partial i/\partial \Phi) = 1$. The maximum current modulation amplitude (Δi) of SQUID sample #1 with inductance $L_s \approx 300$ pH was approximately 3µA.

Branches B_1 and B_2 can be analyzed separately, because they are parallel. First, the function of B_1 (without B_2) is discussed. The magnetic flux in the SQUID-loop ϕ_{Loop} consists of two parts: the external magnetic flux $\Delta\phi_e$ and the additional feedback flux $\Delta\phi_a$ generated by the product of the current change and the mutual inductance M_1. Asymmetric I-Φ characteristic thus results, such as that shown in Figure 1(b). When the slope $\partial i/\partial \Phi$ is positive, the total flux in the loop is changed by the difference of the external and the additional flux, $\Delta\phi_{\text{Loop}} = \Delta\phi_e - \Delta\phi_a$. In case of the negative slope $\partial i/\partial \Phi$, the total flux change is given by the sum $\Delta\phi_{\text{Loop}} = \Delta\phi_e + \Delta\phi_a$. These relations are reversed with the opposite polarity of L_1. The product $\Delta i \cdot M_1$ determines the asymmetry of the I-Φ characteristic. Note that Δi is the modulated current amplitude (swing). If M_1 is adjustable, the critical condition $\Delta i \cdot M_1 = \Phi_0/2$ can be fulfilled, which is approximately expressed as $M_1 \cdot \partial i/\partial \Phi = 1$. In this case, $\Delta\phi_e$ and $\Delta\phi_a$ are equal. It leads to $\Delta\phi_{Loop} = 0$ at the positive slope and $\Delta\phi_{Loop} = 2\Delta\phi_e$ at the negative slope. Accordingly, the flux-to-current transfer coefficient of B_1 is expressed as

$$(\partial i/\partial \Phi)_{B1} = (\partial i/\partial \Phi)/[1-M_1 \cdot (\partial i/\partial \Phi)] \qquad (1)$$

Here, $\partial i/\partial \Phi$ denotes the transfer coefficient of the bare SQUID, without L_1.

In the critical condition $M_1 \cdot \partial i/\partial \Phi = 1$, $(\partial i/\partial \Phi)_{B1}$ reaches infinity at the working point W_2 or $(\partial i/\partial \Phi)/2$ at the working point W_1, see Figure 1(b). In contrast to APF [5], the current swing Δi does not decrease. However, in B_1 not only the symmetry of I-Φ characteristic changes, but also the effective dynamic resistance R_d' (as in APF). In the voltage bias mode, the value of $\partial V/\partial \Phi$ is determined only by SQUID properties, so the relation $\partial V/\partial \Phi = R_d \cdot (\partial i/\partial \Phi) = R_d'_{(B1)} \cdot (\partial i/\partial \Phi)_{B1}$ is still holding. According to (1), $R_d'_{(B1)}$ is

$$R_d'_{(B1)} = R_d \cdot [1-M_1 \cdot (\partial i/\partial \Phi)] \qquad (2)$$

When approaching the critical condition, $R_d'_{(B1)} \rightarrow 0$ at the working point W_2 or $R_d'_{(B1)} \approx 2R_d$ at the working point W_1. Values of R_d and $R_d'_{(B1)}$ were measured by recording the I-V characteristics. The initial R_d of the bare SQUID sample #1 was about 20 Ω. The $R_d'_{(B1)}$ increased to 40 Ω at W_1 while it decreased to 2 Ω at W_2. These values included connection resistances.

Branch B_1 alone does not contribute to the reduction of $\delta\Phi_{preamp.}$, because the product of $R_d'_{(B1)}$ and $(\partial i/\partial \Phi)_{B1}$ remains constant, even though $(\partial i/\partial \Phi)_{W2}$ at W_2 is greatly enhanced.

From here on, we analyze both branches together and take into account the noise from the preamplifier, V_n. The bias voltage can be denoted as V_b+V_n. Here, the dc voltage V_b has been already adjusted to the optimal value, so that the I-Φ characteristic has the maximum amplitude Δi. Note that the bias voltage (V_b+V_n) and the voltage change across SQUID caused by $\Delta\phi_e$ only act as potentials. Therefore, the corresponding voltages are generated by the current $i+i_n$ which always flows from the output of the preamplifier V_m, via R_g to SBC (see Figure 1).

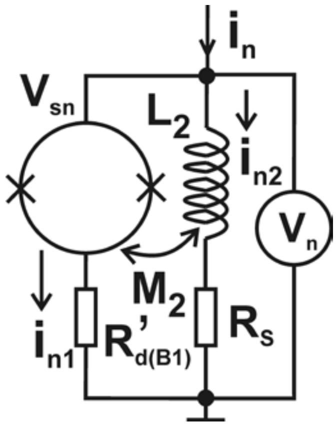

Fig. 2. The noise equivalent circuit of SBC. The noise current i_n flows from the output of the preamplifier into the two parallel branches B_1 and B_2 and generates the noise voltage V_n across each branch.

The noise equivalent circuit of the complete SBC is shown in Figure 2. The following noise analysis applies at low frequencies when $\omega_n L_{1,2} << R_d'_{(B1)}, R_s$. Here, ω_n is the noise frequency. In this circuit, B_1 is represented by the equivalent SQUID having a dynamic resistance $R_d'_{(B1)}$ and a flux-to-current transfer coefficient of $(\partial i/\partial \Phi)_{B1}$; $(\partial V/\partial \Phi)_{B1}$ is the same as that of the bare SQUID. Due to the feedback from B2 the dynamic resistance of the complete SBC, $R_d'_{(SBC)}$, will be higher than $R_d'_{(B1)}$. The total noise current i_n from the output of preamplifier divides into i_{n1} and i_{n2}, which flow in the two branches. The noise voltage across the two branches, V_n, is the same. The current $i_{n2} = V_n/R_s$, if the impedance $\omega_n L_2$ can be neglected. This current will generate a noise flux $i_{n2}\cdot M_2$ into the SQUID-loop, so that a voltage V_{sn} generated by this flux appears across the SQUID:

$$V_{sn} = (V_n/R_s)\cdot M_2\cdot(\partial V/\partial \Phi) \quad (3)$$

According to the relation $V_n = i_{n1}\cdot R_d'_{(B1)} \pm V_{sn}$, the current i_{n1} depends on the voltage ($V_n \pm V_{sn}$). The sign of V_{sn} is determined by the coefficient ($\partial V/\partial \Phi$) and the polarity of L_2. To suppress the preamplifier noise contribution, V_n and V_{sn} should have the same polarity. This is assumed in the discussion below. Considering the relation between V_n and V_{sn}, three different cases are possible:

1) $V_n > V_{sn}$, $i_{n1} = (V_n - V_{sn})/R_d'_{(B1)} > 0$, namely, the current directions of i_n and i_{n1} are the same. There is still a noise current i_{n1} flowing through the branch B_1. The SBC equivalent dynamic resistance $R_d'_{(SBC)}$ is smaller than R_s.

2) $V_n = V_{sn}$, i_{n1} disappears, $i_n = i_{n2}$ and $R_d'_{(SBC)} = R_s$. The noise current $i_n = V_n/R_s$ flows only through B_2, because the equivalent resistance of B_1 for V_n reaches infinity. The condition $V_n = V_{sn}$ is the "critical condition" of B_2. Here, $R_d'_{(B1)}$ does not play a role any more. In this case, shunting a bare SQUID instead of B_1 has the same consequence, namely, $i_n = i_{n2}$. Equality of these currents is attempted in the noise cancellation (NC) technique [6]. As R_s is larger than the R_d of the bare SQUID, the noise current i_n is actually suppressed by a factor R_s/R_d. In our case, branch B_1 of SBC enhances the value of $(\partial i/\partial \Phi)_{B1}$, whereas branch B_2 has no influence on the flux-to-current coefficient so that $(\partial i/\partial \Phi)_{(SBC)}$ is equal to $(\partial i/\partial \Phi)_{B1}$. Therefore, equivalent flux noise of the preamplifier, $\delta\Phi_{preamp.} = V_n/[R_s\cdot(\partial i/\partial \Phi)_{SBC}]$, can be effectively suppressed. Only when $V_n = V_{sn}$, $i_n = i_{n2}$ and $R_d'_{(SBC)} = R_s$, the signal current i in B_1 and the noise current i_n in B_2, are decoupled.

3) $V_n < V_{sn}$, $i_{n1} = (V_n - V_{sn})/R_d'_{(B1)} < 0$. This condition means that the current directions of i_n and i_{n1} are opposite. Here, $R_d'_{(SBC)}$ becomes larger than R_s. In principle, i_n may be reduced down to zero when $R_d'_{(SBC)}$ increases to infinity, but the preamplifier will be no longer acting properly as a pure current-voltage converter.

III. EXPERIMENTS AND RESULTS

A helium-cooled planar SQUID magnetometer (sample #2) with inductance $L_s \approx 350$ pH was used for our first SBC experiments. A pickup loop of 6×6 mm² and a 5 turns input coil are integrated on the SQUID chip. The field-to-flux coefficient $\partial B/\partial \Phi$ was about 0.7 nT/Φ_0. The three wire-wound coils, L_1, L_2 and L_{FLL}, were made from Nb wire and inductively coupled to the SQUID via the pickup loop. The measured I-Φ characteristic of SBC is shown in Figure 3(a); it is significantly asymmetric. By inserting the experimentally observed asymmetry of the transfer function into (1), we obtained the value of $M_1 \approx 0.15$ nH. Measured I-V curves of SBC are shown in Figure 3(b). From the I-V curves shown, we were able to extract $R_d'_{(SBC)}$ values at the two working points, W_1 and W_2. The measured dynamic resistances $R_d'_{(SBC)}$ determine the current noise from the preamplifier, namely $I_n = V_n/R_d'_{(SBC)}$. The initial R_d of the SQUID without SBC was about 9.5 Ω. The measured $R_d'_{(SBC)}$ of about 30 Ω at W_2 is very close to R_s of 29.7 Ω, while $R_d'_{(SBC)}$ at W_1 is reduced to 6 Ω. Therefore, the critical condition of B_2 is approximately fulfilled. The calculated M_2 was about 0.77 nH.

The noise behavior at the two working points shown in Figure 3(a) is quite different. The white flux resolution of the SQUID magnetometer at working point W_2 was measured to be about 4 $\Phi_0/\sqrt{\text{Hz}}$ in flux-locked loop mode, corresponding to a field noise of about 3 fT/$\sqrt{\text{Hz}}$. The noise spectrum is shown in Figure 4. This noise measurement was performed in laboratory environment inside a helium can within a Nb shielding tube. The low frequency noise (< 20 Hz) was mainly caused by mechanical vibrations. The noise was nearly the same as that measured using a standard flux modulation readout circuit. In contrast, the flux noise at the working point W_1 increased to 22 $\Phi_0/\sqrt{\text{Hz}}$.

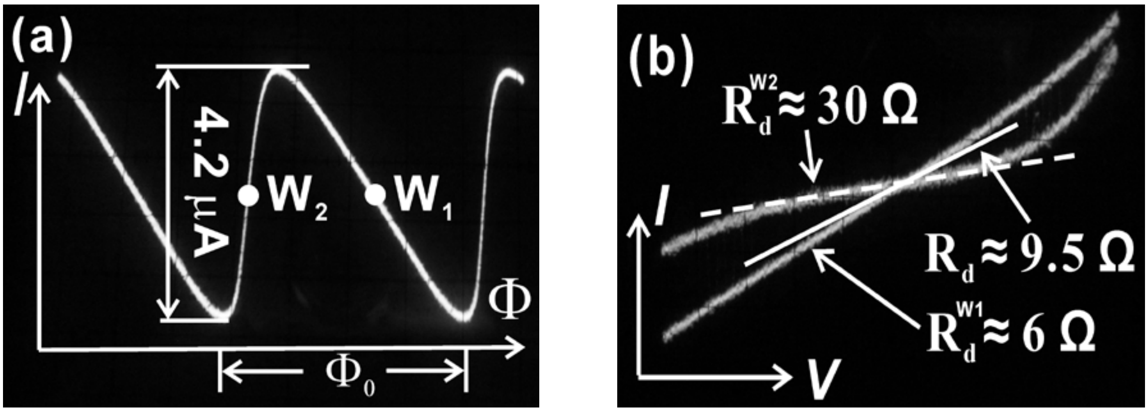

Fig. 3. Measured *I-Φ* (a) and *I-V* (b) characteristics of SBC (SQUID sample #2). The two curves (near the working point) in (b) cross at the two working points W_1 and W_2. The *I-V* curve of the magnetometer without SBC is indicated by the white continuous line.

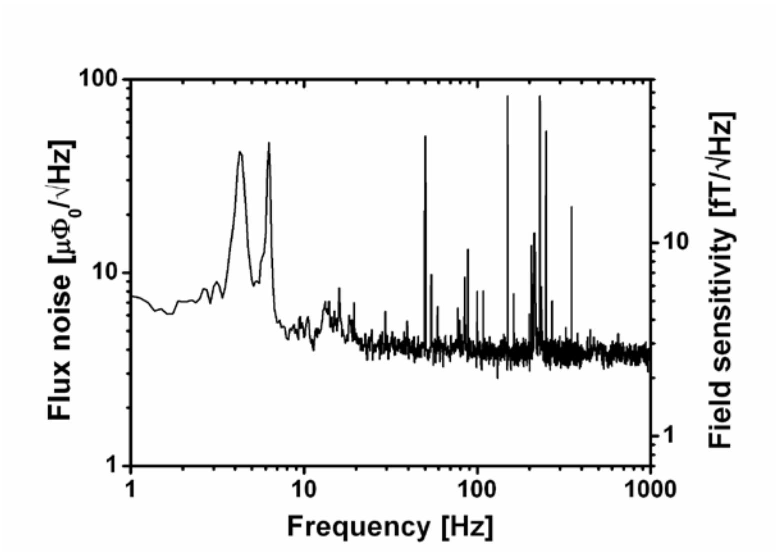

Fig. 4. The measured flux noise spectrum and its corresponding field sensitivity of a helium cooled SQUID magnetometer with a SQUID inductance of 350 pH.

IV. CONCLUSION

We proposed and demonstrated the novel SQUID bootstrap circuit (SBC) to be a very promising electronic scheme for simple low-noise readout of dc SQUIDs in the voltage bias mode. SBC leads to a reduction of the preamplifier noise contribution. The effectiveness of noise suppression is determined by the product of two ratios, $[R_d{'}_{(SBC)}/R_d]$ and $[(\partial i/\partial \Phi)_{SBC} /(\partial i/\partial \Phi)]$. These two ratios can be adjusted separately and each may be changed by a factor between 3 and 5. The adjustment tolerance is thus wider than for either APF or NC. We plan to perform a systematic investigation of SBC to find its optimum parameters. To derive full advantage from the scheme, all additional coils used in SBC should be superconducting and integrated on the SQUID chip. The SBC may become most useful in the SQUID readout/multiplexing of large detector arrays for astrophysics, material analysis, *etc*.

REFERENCES

[1] D. Drung and M. Mück, in *The SQUID Handbook*, J. Clarke and A. I. Braginski, editors (Wiley-VCH, Weinheim), vol. I, pp. 128–155 (2004).
[2] R.L. Forgacs, A.Warnick, *Rev. Sci. Instrum.* **38,** 214 (1967)
[3] F.C. Wellstood, C. Urbina, J. Clarke, *Appl. Phys. Lett.* **50,** 772 (1987).
[4] Y.H. Lee, J.M. Kim, H.C. Kwon *et al*., *IEEE Trans. Appl. Supercond.* **5,** 2156 (1995).
[5] D. Drung, R. Cantor, M. Peters *et al.*, *Appl. Phys. Lett.* **57,** 406 (1990).
[6] M. Kiviranta, H. Seppä, *IEEE Trans. Appl. Supercond.* **5** 2146 (1995).