Magnetic Compensation of Gravity by Using Superconducting Axisymmetric Coils: Spherical Harmonics Method

Clement Lorin, Alain Mailfert
LEM, UMR CNRS n° 7569, INPL, Nancy Université, France
e-mail: Clement.Lorin@ensem.inpl-nancy.fr

Abstract - **It is important for space research to study the behavior of fluids such as liquid oxygen and liquid hydrogen under weightless conditions (microgravity). In addition, since 1991 some magnetic ground-based stations have allowed compensating gravity and meeting space conditions. Magnetic devices allow low-cost microgravity experiments with unlimited time. The goal of these techniques is to reach the same or better conditions (residual acceleration of the studied fluid) than those during parabolic flights. In this paper, several specific distributions of the magnetic field are determined. These distributions allow compensating gravity by means of axisymmetric coils (solenoids). This paper introduces several distributions of the residual forces useful for different kinds of microgravity experiments**.

Submitted November 23, 2007; accepted January 31, 2008. Reference No. ST28, Category 6.
Based on a paper submitted to Proceedings of EUCAS 2007; published in JPCS **98** (2008), paper # 012199

I. INTRODUCTION

The sum of gravity and centrifugal forces is zero for a spacecraft, such as an artificial satellite, space station or spaceship maintaining a stationary orbit with the power unit stopped (when no impulse is acting on the spacecraft). When the spacecraft is not deformable and has no own kinetic moment, all its constituents are subjected to zero acceleration. This is the state of weightlessness. However, at the time of space flight, various phenomena, such as the rotation of the spacecraft, emission from a field emitter or the motion of an astronaut, can induce residual acceleration. The term “micro-gravity” is used when the residual acceleration is less than 10^{-2}g.

The study of the material behaviour, in particular the fluids, under weightless conditions can be carried out by means of satellites or space stations, but is easier attained with parabolic flights, sounding rockets or drop towers. The magnetic levitation technique, which is still cheaper, allows one to counterbalance the gravity. This counterbalance holds for individual molecules constituting the levitating material, which amounts to simulating the space conditions on the Earth.

In 1991, Beaugnon and Tournier succeeded in magnetic levitation of water, graphite and some organic materials such as wood, plastic, ethanol, etc [1]. These experiments opened the way to other, especially those helping to understand the cryogenic fluid behaviour, such as liquid hydrogen or liquid oxygen, under weightless conditions [2, 3]. Among others things, the knowledge of the weightless fluid behaviour makes possible both development and improvement of booster rockets.

The magnetic force vector $\vec{G}$, specific to magnetic field distribution and introduced in Section II below, has an accurate value for each material. Table 1 provides some values of this quantity.

Table 1. Values of G for some materials

Substance	$\vec{G}=\overrightarrow{grad}B^2$ (T^2/m)
O_2(90K)	8
NO (118K)	516
H_2 (20K)	-986
H_2O (293K)	-2717
He (5K)	-4174
N_2 (78K)	-4578
Xe (293K)	-5661

These values of $\vec{G}$ can be technically obtained in a 3D region of volume rapidly increasing with magnetic field and decreasing with increasing homogeneity of the compensation. Magnetic levitation does not allow us to obtain full gravity compensation in a 3D domain. Therefore, there exists a residual acceleration during magnetic levitation experiments. This acceleration can be approximately calculated [4]. For example, in order to levitate one cubic centimetre of liquid hydrogen at 20K with an inhomogeneity less than 1%, *i.e.*, a residual acceleration less than 10^{-2}g, it is necessary to use a 12T superconducting electromagnet.

Magnetic levitation experiments are always carried out within simple-layout solenoids, sometimes improved by ferromagnetic inserts. The present paper discusses more complicated magnetic field sources consisting of several windings, like in NMR coils. These sources allow one to attain various perfectly known residual acceleration configurations. The order of magnitude of the magnetic levitation homogeneity is similar to that of other facilities for carrying out experiments under weightless conditions.

II. MAGNETIC FORCE AND INHOMOGENEITY VECTOR

A magnetic field exerts a force density proportional to $\vec{\nabla}B^2=\vec{G}$**,** on weakly magnetic materials (dia- and para-magnetic materials) in vacuum, expressed as:

$$\frac{d\vec{f}}{dV}=\frac{1}{2\mu_0}.\chi_m.\vec{\nabla}B^2, \quad (1)$$

where $\frac{d\vec{f}}{dV}$ is the magnetic force density (N/m^3), μ_0 the vacuum permeability (H/m), χ_m the magnetic susceptibility (dimensionless), $\vec{B}$ the magnetic flux density (T).

A magnetic force density cannot be constant in a 3D domain, thus a perfect compensation of gravity is unreachable in a 3D region of space using only magnetic fields. This has been demonstrated in previous work [5].

The relative error between perfect compensation $\vec{G}_1$ and the effective compensation $\vec{G}$ at the considered point is defined by the inhomogeneity vector $\vec{\varepsilon}$:

$$\vec{\varepsilon}=\frac{\vec{G}-\vec{G}_1}{\left|\vec{G}_1\right|} \quad (2)$$

$$\vec{G}_1 = \frac{-2.\mu_0.\rho}{\chi_m}\vec{g} \tag{3}$$

where $\vec{g}$ = 9.81 m.s^{-2} is the terrestrial acceleration, and ρ the density (kg/m^3).

In this paper, a method for determining the $\vec{G}$ field is developed; this method is of interest for the case of axisymmetric geometry (solenoidal coils). The method was first used by Garrett [7] [8] in order to obtain very uniform fields within solenoidal systems (NMR coils). This method starts with a harmonic decomposition of the scalar magnetic potential V in the useful zone assumed without currents (resolution of the Laplace equation). This suggests use of a spherical harmonic decomposition of the magnetic field. In this paper, the value of $\vec{G}$ is calculated according to these field harmonics.

The configurations of the inhomogeneity vector $\vec{\varepsilon}$, depending on the desired conditions of micro-gravity, can be expressed by the setting of some derivatives of the vector $\vec{G}$ equal to zero. These conditions allow calculating the field harmonics. To conclude; the determination of the corresponding field sources is obtained by resolution of the inverse problem of the magneto-statics, leading to determination of the spatial harmonic of the currents providing the desired fields.

III. CALCULATING METHOD

A. Definition of the Geometry

The spherical coordinates can be reduced to the (r,θ) couple for an axisymmetric system. The symmetry axis is defined by θ = 0 in Figure1. The x-axis direction is chosen opposite to the gravity vector $\vec{g}$:

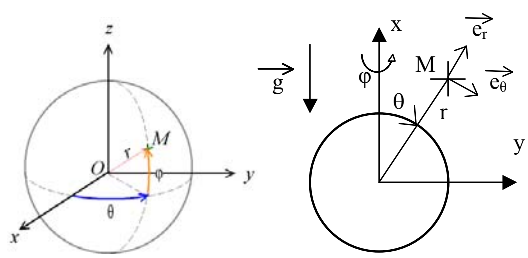


Fig. 1. Spherical coordinates and geometry of the system

B. Vector $\vec{G}$ at any Point in the Sphere

The distributions of the field in a sphere of radius R_0 , centred on O and called the working zone, are studied. The axisymmetric current sources are assumed to be either on the surface or outside of the working zone. An infinite number of current distributions can create the same magnetic field distribution within the working zone. In order to calculate the exact solutions of the magnetic field at any point of the space, the current sources are assumed here to be made up of a surface current layer on the radius R_0 sphere. One assumes the norm of the magnetic field at the centre to be equal to B_1 :

$$\left|\vec{B}(0,\theta)\right| = \left|\vec{B}_1\right|$$

Previous work has demonstrated that the magnetic field has to be as high as possible to obtain the best magnetic compensation homogeneity [5]. This suggests the use of superconducting coils with a very high field. At the centre of the working zone (r=0), the value G_1 of the norm of $\vec{G}$ perfectly counterbalances gravity for the considered fluid [5]:

$\left|\vec{G}(0,\theta)\right|=\left|\vec{G}_1\right|$

The solution of the Laplace equation for the magnetic scalar potential, in spherical coordinates, leads to the components of the magnetic field. The boundary conditions provide two different solutions, a first one inside the sphere and a second one outside the sphere.

The inner and outer magnetic fields can be expressed by the Legendre polynomials P_n. The components of the n-th field harmonics, along the vectors $\vec{e}_r$ and $\vec{e}_\theta$, are :

$$\overrightarrow{H_{int\,n}}=\begin{vmatrix} -n.C_n.r^{n-1}.P_n(cos\theta) \\ -C_n.r^{n-1}.P_n^1(cos\theta) \end{vmatrix} \qquad r<R_0 \tag{4}$$

$$\overrightarrow{H_{ext\,n}}=\begin{vmatrix} (n+1).\dfrac{C'_n}{r^{n+2}}.P_n(cos\theta) \\ -\dfrac{C'_n}{r^{n+2}}.P_n^1(cos\theta) \end{vmatrix} \qquad r>R_0 \tag{5}$$

The coefficients C_n will be calculated from the homogeneity conditions of the inner quantities. The continuity conditions allow determining C'_n and the surface current density. The magnetic field expression (4) within the working zone can be obtained by an infinite number of current distributions, but the two relations (4) and (5) together are true only for surface current density on the R_0 radius sphere. Relation (4) involves :

$$B_{int}{}^2=\mu_0{}^2\sum_{n=1}^{\infty}n.C_n.r^{n-1}.P_n(cos\theta).\sum_{p=1}^{\infty}p.C_p.r^{p-1}.P_p(cos\theta)+\mu_0{}^2\sum_{n=1}^{\infty}C_n.r^{n-1}.P_n^1(cos\theta).\sum_{p=1}^{\infty}C_p.r^{p-1}.P_p^1(cos\theta) \tag{6}$$

The vector $\vec{G}$ is derived from this quantity :

$$\vec{G}(r,\theta)=2\mu_0{}^2\begin{bmatrix} \left[\sum_{n=1}^{\infty}n.C_n.r^{n-1}.P_n(cos\theta)\sum_{p=1}^{\infty}p.(p-1).C_p.r^{p-2}.P_p(cos\theta)\right]+\left[\sum_{n=1}^{\infty}C_n.r^{n-1}.\dot{P}_n(cos\theta).\sum_{p=1}^{\infty}(p-1).C_p.r^{p-2}\dot{P}_p(cos\theta)\right] \\ \left[\sum_{n=1}^{\infty}n.C_n.r^{n-1}.P_n(cos\theta).\sum_{p=1}^{\infty}p.C_p.r^{p-2}.\dot{P}_p(cos\theta)\right]+\left[\sum_{n=1}^{\infty}C_n.r^{n-1}.\dot{P}_n(cos\theta).\sum_{p=1}^{\infty}p.C_p.r^{p-2}.\ddot{P}_p(cos\theta)\right] \end{bmatrix} \tag{7}$$

Any distribution of magnetic forces in a spherical free space cavity (without current) can be expressed by the expression (7), namely by the coefficients C_n.

C. Magneto-gravitarian Potential

The inhomogeneity vector is derived from a « magneto-gravitarian » potential Σ_L (in meter) defined from the expression (2) :

$$\Sigma_L=\frac{\left|\vec{B}\right|^2}{G_1}-z \tag{8}$$

where $\vec{B}$ is the magnetic flux density (T), G_1 the norm of the gradient allowing the levitation of the considered material [4] (T^2/m), and z the height (m). If a static fluid, near its critical point, *i.e.*, with a surface tension close to zero, is subjected only to gravity and the magnetic force, then its free surface must be given by the equipotentials « isoΣ_L ». The new solution of the problem, dealing with the magnetic compensation of gravity, by the potential Σ_L provides interesting results an example of which is given below.

D. Choice of the Homogeneity Conditions and Residual Forces

The conditions on the homogeneities define the values of the field harmonics $(C_i)_{i\in N}$ by the conditions imposed on the n-th derivatives of the vector $\vec{G}$. Three distinct conditions are examined. Each one describes a specific inhomogeneity (resulting accelerations are symbolized by arrows in Figure 2) and leads to interesting experimental conditions of micro-gravity. The residual acceleration vector is either orthoaxial, that is central in a plane perpendicular to the symmetry axis (a), or orthogonal to the *yOz* plane (b), or central (c). The equipotentials Σ_L are respectively cylinders centred on the *Oz* axis, or a plane perpendicular to the *Oz* axis, or spheres centred on *O*.

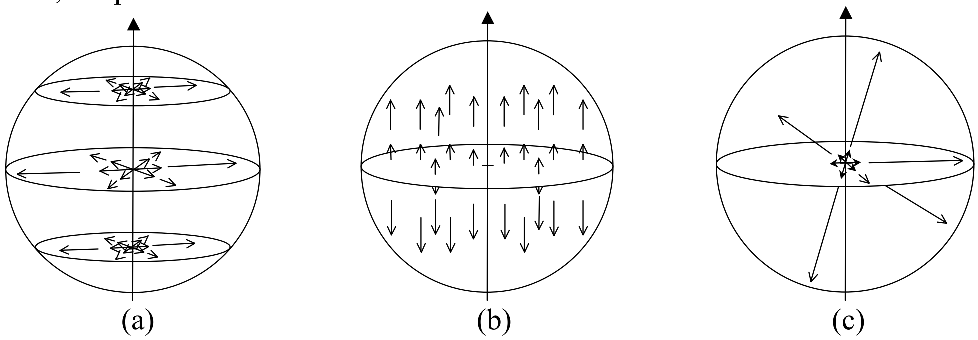


Fig. 2. Distributions of the residual accelerations symbolized by arrows.

- In the orthoaxial case, $\varepsilon_r(r,0)$ tends towards zero, and requires :

$$\forall n \in N \;,\; \left.\frac{\partial^n Gr(r,0)}{\partial r^n}\right|_{r=0} = 0 \tag{9}$$

where $\forall n \in N$ *means : for each n that is an element of the natural number set N.*

- In the orthogonal case, $\varepsilon_r(r,\frac{\pi}{2})$ and $\varepsilon_\theta(r,\frac{\pi}{2})$ tend towards zero :

$$\forall n \in N \;,\; \left.\frac{\partial^n Gr(r,\frac{\pi}{2})}{\partial r^n}\right|_{r=0} = \left.\frac{\partial^n G\theta(r,\frac{\pi}{2})}{\partial r^n}\right|_{r=0} = 0 \tag{10}$$

- In the central case, $\varepsilon_r(r,\frac{\pi}{2})$ and $\varepsilon_r(r,0)$ must be equal :

$$\forall n \in N \;,\; \left.\frac{\partial^n Gr(r,0)}{\partial r^n}\right|_{r=0} = \left.\frac{\partial^n Gr(r,\frac{\pi}{2})}{\partial r^n}\right|_{r=0} \tag{11}$$

Equations (8), (9) and (10) allow one to calculate the values of the coefficients of the field harmonics $(C_i)_{i\in N}$, these values are given in Table 2.

Table 2. Values of the first six harmonics of the field for the three configurations

	C_1	C_2	C_3	C_4	C_5	C_6
Orthoaxial	$\frac{B_1}{\mu_0}$	$\frac{G_1}{4\mu_0 B_1}$	$\frac{-G_1^2}{24\mu_0 B_1^3}$	$\frac{G_1^3}{64\mu_0 B_1^5}$	$\frac{-G_1^4}{128\mu_0 B_1^7}$	$\frac{7G_1^5}{1536\mu_0 B_1^9}$
Orthogonal	$\frac{B_1}{\mu_0}$	$\frac{G_1}{4\mu_0 B_1}$	$\frac{G_1^2}{48\mu_0 B_1^3}$	0	$\frac{-G_1^4}{3840\mu_0 B_1^7}$	$\frac{-G_1^5}{46080\mu_0 B_1^9}$
Central	$\frac{B_1}{\mu_0}$	$\frac{G_1}{4\mu_0 B_1}$	$\frac{-G_1^2}{48\mu_0 B_1^3}$	$\frac{G_1^3}{128\mu_0 B_1^5}$	$\frac{-41G_1^4}{6400\mu_0 B_1^7}$	$\frac{23G_1^5}{7680\mu_0 B_1^9}$

It can be noted that the first two harmonics are the same in all cases. If the harmonic orders higher than two are zero, a fourth configuration appears in which the magneto-gravitarian equipotentials are spheroids with an eccentricity equal to two.

E. Calculation of the Surface Current Density

The determination of the currents from the established distributions of the field is an inverse problem in magnetism, with an infinity of solutions. The easiest theoretical solution is the surface current distribution on a sphere of radius R_0 . In this case, the coefficients of relations (4) and (5) are linked as follow :

$$C'_n = -\frac{n}{n+1}.R_0^{2n+1}.C_n \tag{12}$$

According to relations (4), (5), (12) and the equality between the tangential components of the field and the currents, the surface current density harmonics are expressed as :

$$\overrightarrow{K_n} = \frac{2n+1}{n+1}.R_0^{n-1}.C_n.P_n^1(\cos\theta)\overrightarrow{e_\varphi} \tag{13}$$

IV. RESULTS OF THE NUMERICAL SIMULATION

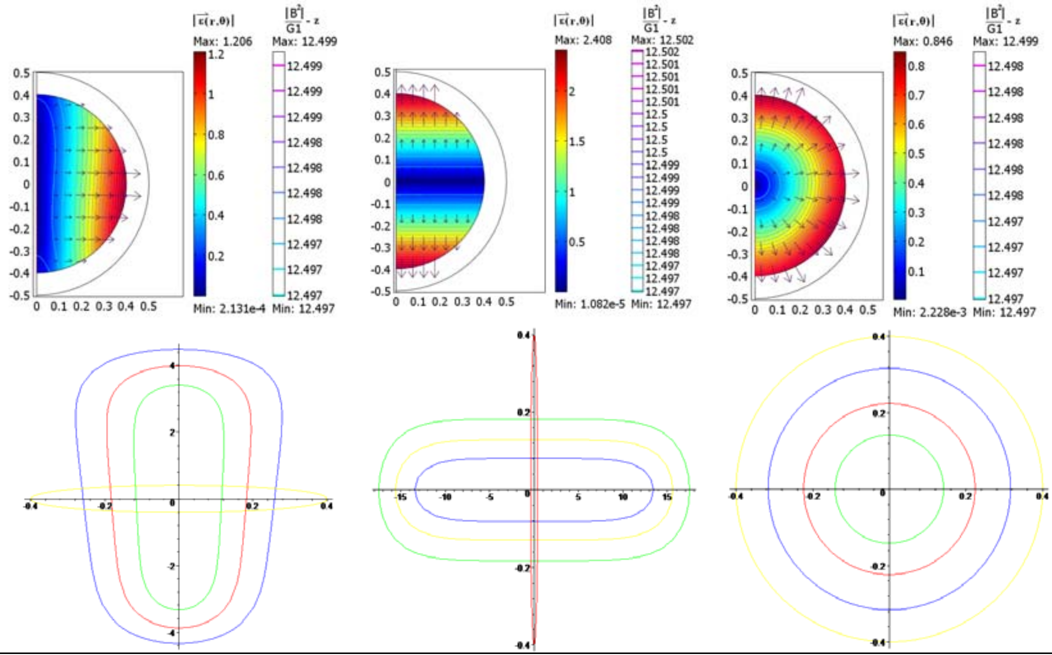
A numerical simulation of the different configurations of micro-gravity is possible by choosing the sources of current given by relation (13), with the values of the coefficients C_n of Table 2. In our simulation, the surface current density is arbitrarily truncated at the sixth harmonic.

The simulations are carried out on liquid oxygen at 90K, with a gradient G_1=8T^2/m and a magnetic field at the origin of B_1=10T. The surface current density, previously obtained, is spread over a 0.5 meter radius sphere.

The figures in the first line of Table 3 are obtained with a finite element software. Inside a 0.4 meter sphere, the black arrows represent the vector $\vec{\varepsilon}$, in fact the residual acceleration. The norm of the inhomogeneity (in %) is provided by the colour bar to the right of each figure. The bluish lines are the isoΣ_L (in meters). These isoΣ_L are also plotted in the second line of Table 3, according to an analytical calculation taking into account only the first six harmonics. In each figure of the second line is drawn a 0.4 radius circle because the scales are not normed.

Table 3. Representation of the three configurations with only the first six harmonics

Orthoaxial	Orthogonal	Central



According to Tables (2) and (3), the third harmonic seems to fix the anisotropy of the residual acceleration. The first three harmonics make it possible to fix, the magnetic field and the gradient at the centre, and the resulting acceleration vector, respectively.

V. CONCLUSION

The method, briefly described in this paper, partially uses previous work of our team [6]. This method provides elements of vital importance for the development of a magnetic levitation device. The practical design of the superconducting coils needed to create the desired magnetic fields is not introduced in this paper but will be the subject of further studies. The design of these devices uses the same methods as those employed for the superconducting coils of NMR systems.

This work introduced the useful concept of the magneto-gravitarian potential describing the residual forces when the magnetic compensation of the gravity is carried out. Various distributions of the resulting forces attainable by this method are possible. A good choice of the harmonic coefficients previously defined allows one to adapt the distributions. The various choices of the coefficients allow one to carry out a wide range of experiments in the ground-based simulation station of microgravity. The three examples introduced above lead to kinds of experiments interesting for the study of fluids or granular matter in space conditions.

This general method, succinctly developed here, can be adapted to other levitation devices, in a cylindrical geometry [4] for example. In this case, these devices could be built from multipoles as for particle accelerators: mainly dipoles, quadrupoles and sextupoles.

ACKNOWLEDGEMENTS

The authors thank Daniel BEYSENS of the ESPCI Paris, Denis CHATAIN of the CEA Grenoble and Lionel QUETTIER of the CEA Saclay for some beneficial and useful discussions,

REFERENCES

[1] Beaugnon E. and Tournier R., Levitation of organic materials *Nature* **349**, 470 (1991).

[2] Wunenberg R., Chatain D., Garrabos Y. and Beysens D., Magnetic compensation of gravity forces in (-p) hydrogen near critical point : Applications to weightless conditions *Physical Review* E **62**, 469-476 (2000).

[3] Chatain D. and Nikolayev V. S., Using magnetic levitation to produce cryogenic targets for inertial fusion energy: experiment and theory *Cryogenics* **42**, 253-261 (2002)

[4] Quettier L., Félice H., Mailfert A., Chatain D. and Beysens D., Magnetic compensation of gravity forces in liquid/gas mixtures: surpassing intrinsic limitations of a superconducting magnet by using ferromagnetic inserts *The European Physical Journal Applied Physics* **32**, 167-175 (2005).

[5] Lorin C. and Mailfert A., Magnetic compensation of gravity, and centrifugal forces *ELGRA 2007 conference, Florencia* submitted to *Microgravity Science and Technology* (2007).

[6] Quettier L., Contribution méthodologique à la conception de systèmes supraconducteurs de lévitation magnétique *Thesis INPL* (2003).

[7] Garett M. W., Thick cylindrical coil systems for strong magnetic fields with field or gradient homogeneities of the 6^{th} to 20^{th} order *Journal of Applied Physics* **38**, 6, 2563-2586 (1967).

[8] Garett M. W. Axially symmetric systems for generating and measuring magnetic fields *Journal of Applied Physics* **22**, 9, 1091-1107 (1951).