Relation Between Transverse and Longitudinal Normal Zone Propagation Velocities in Impregnated MgB₂ Windings

Antti Stenvall, Risto Mikkonen, and Pavol Kovác

Abstract—The transverse normal zone propagation velocity, \( v_t \), in impregnated magnets controls the 3D normal zone expansion during a quench. It is dominated by the thermal conductivities of the conductor insulation and the impregnation material. The longitudinal propagation velocity \( v_l \) is mainly determined by the heat generation, critical surface of the superconductor and thermal conduction along the conductor. It has been generally assumed that the ratio \( v_t/v_l \) is proportional to the square root of the ratios of the corresponding effective heat conductivities. In this paper we study computationally the validity of this approach for an MgB₂ wire surrounded by an epoxy layer. We take into account the finite \( n \)-value of the composite conductor in our Finite Element Method (FEM) models. We computed \( v_t \) with Whetstone-Roos formula and 1D and 2D FEM models. The 2D model was also used to compute \( v_l \). In addition to this, minimum quench energies given by the 1D and 2D FEM models were compared.

Index Terms—finite element method, MgB₂, normal zone propagation velocity, simulation

I. INTRODUCTION

TRANSVERSE normal zone propagation velocity \( v_t \) dominates the quench behaviour of an adiabatic superconducting magnet. It is largely dependent on the heat conductivity of the impregnation material and conductor insulation. The measurement of longitudinal normal zone propagation velocity \( v_l \) is essentially easier to measure than \( v_t \), and also computation of it can be performed easily numerically or by utilizing some of the analytical formulae [1]–[3].

Some quench programs, such as Wilson’s QUENCH [4, ch.9], utilize propagation velocities to compute the normal zone expansion in the winding, \( v_l \) can be approximated from \( v_l \) by applying the ratios of transverse and longitudinal heat conductivities \( \lambda_t \) and \( \lambda_l \) respectively as [4, p.208]

\[
v_l \approx v_l \sqrt{\frac{\lambda_l}{\lambda_t}}
\]

(1)

Other analytical approaches exist too [5, p.353], whereas Finite Element Method (FEM) has been applied also earlier to solve \( v_l \) for an NbTi winding [6]. Term \( \sqrt{\frac{\lambda_t}{\lambda_l}} \) is defined \( \alpha \) and \( v_t/v_l = \beta \). \( \beta \) depends largely on the coil filling factor, but for Nb-based conductors estimates between 0.1-0.01 have been presented [7].

By knowing \( \beta \) and \( v_l \) reliably, it is possible to use with enough accuracy a quench simulation program which utilises normal zone propagation velocities. This kind of programs are faster than quench simulation programs which discretise the winding and adopt, e.g., FEM of finite difference method [8]–[10].

In this paper we study computationally if \( \alpha \) can be used to estimate \( \beta \) and derive \( v_t \) from \( v_l \) in MgB₂ windings with finite \( n \)-value. We derive \( v_t \) values from the Whetstone-Roos formula (W-R) [3] and 1D and 2D FEM models. For quench initiation in FEM models we compute the minimum quench energies (MQE) and ignite quenches with energies 10% higher than MQE. \( v_l \) values are also computed with 2D FEM model and derived from the \( v_l \) values by using \( \alpha \). We did computations with two \( n \)-values, 15 and 30.

II. COMPUTATIONAL MODEL

Quench onset has been studied by solving the heat diffusion equation in transient conditions [11]–[13]. It has been noted that traditional closed form approaches are not applicable when solving minimum quench energies of MgB₂ conductors [12]–[14]. This is especially due to finite \( n \)-values. We proposed a model for computing minimum propagation zones in [15] and adopted it to study also normal zone propagation velocities \( v_{NZP} \) in [16]. The results given by the model were also compared with measurements of multifilamentary MgB₂ wire with relatively good accuracy. Here we briefly review the model and present it in applicable form for computation of both minimum quench energies and \( v_{NZP} \).

The basis of the model is to solve with FEM software the heat diffusion equation

\[
\nabla \cdot \lambda \nabla T + Q + Q_{ext} = C_p \frac{\partial T}{\partial t},
\]

(2)

where, \( T \), \( Q \), \( Q_{ext} \) and \( C_p \) are the temperature, the Joule heat generation, the external heat generation and the volumetric specific heat, respectively.

In our approach, heat generation was computed according to the power-law as

\[
Q = \frac{I}{A} \times \min \left\{ E_c \left( \frac{I}{I_c(B,T)} \right)^n, \rho_{norm}(T) \frac{I}{A} \right\}
\]

(3)

where, \( I \), \( A \), \( E_c \), \( I_c \), \( B \) and \( \rho_{norm} \) are the operation current, the conductor cross-sectional area, the electric field criterion (here 1 \( \mu \)V/cm), the critical current, the magnetic flux density and the conductor normal state resistivity, respectively. It is
important to note that this model includes current sharing because $n$-values are measured for entire conductors to characterize the current-voltage relation. Furthermore, it is not reasonable to use bulk, or single crystal, $n$-values to give $E-I$ relation for superconducting region of the conductor because then the extrinsic defects during the wire manufacturing are not included. These are, e.g., local variation of critical current density, sausaging and grain boundaries. After all, it has been discussed that conductor $n$-values arise from the local variation of cross-sectional critical currents [17].

External heat input can be computed in several ways. We used rectangular pulse with fixed duration and thus $Q_{ext}$ was determined as

$$Q_{ext} = \begin{cases} 0 & \text{when } t < 0 \text{ or } t \geq t_{dist} \\ \frac{E_{ext}}{t_{dist}} & \text{when } 0 \leq t < t_{dist} \end{cases}, \quad (4)$$

where $t_{dist}$ and $E_{ext}$ are the disturbance duration and its energy, respectively. We had always $t_{dist} = 10$ ms.

Minimum quench energy $E_{MQE}$ was determined as when $E_{ext} = E_{MQE}$ it states that

$$T (\text{Model's symmetry point})|_{t = t_{test}} = T_{cs}, \quad (5)$$

where $t_{test}$ is the test time used in the numerical algorithm and $T_{cs}$ is the current sharing temperature which is derived as the inverse function of $I_c(T)$.

From the solved $T(t)$ distributions values of $v_{NZP}$ were determined as

$$v_{NZP} = \frac{1}{N-1} \sum_{i=1}^{N-1} \frac{||x_{i+1} - x_i||}{t(T(x_{i+1}) = T_{cs}) - t(T(x_i) = T_{cs})}, \quad (6)$$

where $x_i$s are points in the model. Fig. 1 shows schematically how we defined in the 1D FEM model $v_{t}$ values. Also symmetry point for determination of MQE is shown. In Fig. 2 corresponding 2D model is shown with a view for determination of $v_{t}$.

Normal zone propagation velocities derived from the solutions were compared with W-R:

$$v_{W-R} = \frac{I}{A} \int \frac{1}{\rho_{norm}(T_{cs}) \lambda_1(T_{cs})} \times \left( C_p(T_{cs}) - \frac{1}{\lambda_1(T_{cs})} \frac{d\lambda_1(T_{cs})}{dT} |_{T = T_{cs}} \right)^{\frac{1}{2}} \int_{T_{op}}^{T_{cs}} C_p(T)dT \right), \quad (7)$$

where $T_{op}$ is the operation temperature.

### III. MgB$_2$ Wire

The study was done with an MgB$_2$/Ti/Cu/Monel wire (Fig. 3) with volumetric fractions of 7% MgB$_2$, 14% Ti, 24% Cu and 55% Monel. Details of the conductor were given in [18].

Figure 4 presents critical current characteristics for investigated wire with selected values of magnetic flux density. Material properties for the simulations were collected from the literature (see Table I). Not all the material properties were available and we used stainless-steel (SS) instead. RRR value of copper was expected to be 100. Effective material properties for the wire were computed according to the formulae given
Fig. 4. Critical current characteristics for investigated wire.

Table I

<table>
<thead>
<tr>
<th>Material</th>
<th>(\rho)</th>
<th>(C_p)</th>
<th>(\lambda)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MgB(_2)</td>
<td>[21]</td>
<td>[22]</td>
<td>[21]</td>
</tr>
<tr>
<td>Ti SS</td>
<td>[23, X-Z-4]</td>
<td>[23, VIII-O-3.1]</td>
<td>[24, 12-137]</td>
</tr>
<tr>
<td>Cu</td>
<td>[25, X-E-5]</td>
<td>[23, VIII-B-1]</td>
<td>[23, VII-B-2.1]</td>
</tr>
<tr>
<td>Epoxy</td>
<td>-</td>
<td>[25]</td>
<td></td>
</tr>
</tbody>
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in [19]. For MgB\(_2\) \(C_p\), we expected porosity of 50% based on the studies presented in [20].

IV. RESULTS

We chose to study the presented wire at 10 K and 3 T. Then \(I_c\) was 147 A. In the 2D model we used epoxy thickness of 1 mm. We chose \(t_{\text{test}} = 2\) s. Fig. 5 shows the modelled temperature at symmetry point at 10 K and 3 T as a function of disturbance energy for few times when \(I = 0.7I_c\). Here 1D model was used. This shows that our choice was adequate. However, when \(n\)-value was 15 and \(I\) approached \(I_c\), the sample warming was high enough and \(T_{cs}\) was reached before \(t = 2\) s even without any disturbance in our 1D model. Due to this we stopped our simulations to \(t = 0\).

For the starting point of the \(v_l\) analysis we needed to compute MQE (Fig. 6). Based on our earlier work, it was known that low \(n\)-values will result in higher MQE than high ones [15]. However, it is notable that the 2D FEM model gave considerably higher values for MQE than the 1D model. This hints that MQE measurements done for short-samples are very pessimistic when considering MQE of coils if they operate considerably below the critical current. For example at \(0.5I_c\) and \(n = 15\) 2D model gave sevenfold MQE. However, when \(I_c\) was approached the stabilisation given by the transverse heat conduction played smaller role. At 0.9\(I_c\) the sevenfold MQE diminished to twofold.

Fig. 7 presents the computed values of \(v_l\). W-R agreed better with the results derived from the 1D model and with \(n = 30\). However, the 2D model gave considerably lower propagation velocities than the 1D model and also when \(I_c\) was approached \(v_{W-R}\) increased much more rapidly than the FEM models predicted. At 0.5\(I_c\) and \(n = 30\) 1D model agreed best with W-R. Still the difference was 30% (5 cm/s). Corresponding difference in case of \(n = 15\) was more than 40%. When \(I = 0.9I_c\) the \(v_l\) values given by 1D model were 80% and 150% higher for \(n\)-values 30 and 15, respectively, than \(v_{W-R}\).

When the differences in \(v_l\) were transferred to \(v_l\) by \(\alpha\), computed at \(T_{cs}\), they remained almost constant as seen in Fig. 8. When we look at 2D model, \(\alpha\) estimated \(\beta\) with good accuracy at low currents but when current reached 0.8\(I_c\) the error is already more than 50% and at 0.9\(I_c\) it is more than 100%.
Fig. 7. Longitudinal normal zone propagation velocities.

Fig. 8. Transverse normal zone propagation velocities.

V. CONCLUSIONS

We presented 1D and 2D models to compute longitudinal and transverse normal zone propagation velocities. We compared the results achieved with our models to the Whetstone-Roos formula and found the best agreement with 1D FEM model and W-R at low currents and high \( n \)-value. However, the correspondences of both 1D models with 2D FEM model were bad. This was seen also when MQE values were computed. It was also shown that in this particular case it is not adequate to estimate transverse normal zone propagation velocity from the longitudinal with the square root of the ratios of corresponding effective heat conductivities.

REFERENCES