electron mean free path l_{Al} in Al interlayers. In the bulk aluminum l_{Al} is the constant ~ 120 nm whereas in the films l_{Al} increases with film thickness and is of the order of d_{Al} since it is mainly limited by the surface scattering [14]. Thus, for $d_{Al} \geq$ 100 nm we have an intermediate situation between dirty and

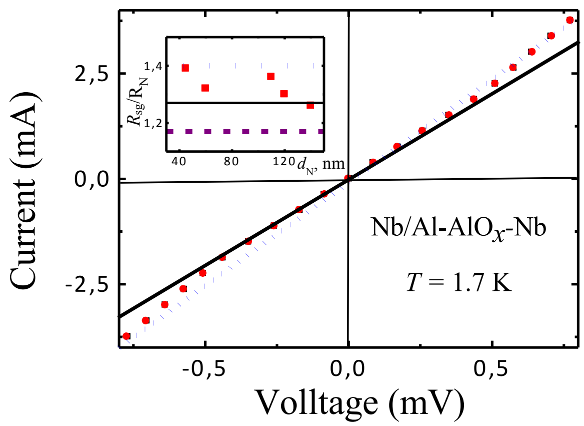

Fig. 2. The main panel: dissipative current–voltage characteristic of a representative Nb/Al–AlO$_x$–Nb junction with a 140 nm-thick Al interlayer at 1.7 K (squares); solid and dotted straight lines correspond to Ohm's laws with R_{sg} = 0.34 Ohm and R_N = 0.27 Ohm. The inset shows R_{sg}/R_N values for Al films with different thicknesses d_N; solid, dotted, and dashed lines correspond to theoretical values for S/N-I-S (quasi-clean limit, d_N/= l_N, d_N/= $2\xi_N$*, this paper), N-I-S, and S/N-I-S (dirty limit with proximity-effect taken into account [8]), respectively.

clean limits. To take it into account, we have introduced an additional imaginary term $i/(2l_{Al})$ in the wave vector $k^{e(h)}$ of an electron (a hole) in the Al interlayer (see [15]).

The charge-transport problem in S/N-I-S junctions can be regarded as one-dimensional since the presence of the potential barrier drastically reduces the tunneling probability with increasing the incidence angle. We have calculated *I-V* curves for S/N-I-S four-layered junctions with arbitrary transparency *D* of the interspace between two superconducting electrodes. Our numerical simulations at zero temperature repeated in general terms similar calculations developed earlier [16-19] with the only exception, the presence of an additional phase shift originated from two passages across the N layer, first, as an electron (a hole) with an energy *E* and a wave vector $k^{e(h)}$ and, second, as a hole (an electron) after Andreev electron-into-hole (hole-into-electron) transformation. Related phase shifts are $\varphi^e = k^e d_N$ and $\varphi^h = -k^h d_N$ (note that the hole is moving in the direction opposite to that of its wave vector). Adding the phases we get the following relation for an additional phase shift induced by the presence of an N interlayer

$$\varphi = \left(k^e - k^h\right) d_N = 2\varepsilon d_N / \left(\hbar v_F\right) + i d_N / l_N \approx (\varepsilon / \Delta) d_N / \xi_N{}^* + i d_N / l_N$$

here $\varepsilon = E - E_F$, v_F is the Fermi velocity. The proximized Al interlayer changes the probability amplitude of an Andreev-scattering process $\chi^{eh(he)}(\varepsilon) = -\arccos\left(\varepsilon / \Delta\right)$ in a conventional s-wave superconductor [13] to $\tilde{\chi}^{eh(he)}(\varepsilon) = \varphi - \arccos\left(\varepsilon / \Delta\right)$ and it is just the effect we are looking for.

The total quasiparticle current-vs-voltage characteristics of S/N-I-S heterostructures can be represented as a sum of independent contributions from individual transverse modes with a known distribution $\rho(D)$ of their transmission probabilities. Averaging the *I-V* curves for fixed parameters Z with the distribution function (1) we get an expected dissipative current-voltage characteristic for an S/N-I-S four-layered device with ideally disordered insulating barrier. Further, it will be characterized by the ratio of the subgap resistance $R_{sg}(\tilde{V}) = \tilde{V} / I_{qp}(\tilde{V})$ calculated at very low voltages $\tilde{V}$, to the normal-state resistance $R_N = 1/\bar{G}$ of the junction. Our numerical result $R_{sg}(\tilde{V}) / R_N = 1.27$ should be compared with related experimental data.

Measurements of *I-V* characteristics at 1.7 K have been carried out suppressing the critical current of the junctions with applied magnetic field, in an experimental apparatus already described in the previous paper [12]. The S/N-I-S junctions were fabricated as was reported earlier in Refs. 6-8. In order to test the new theoretical approach described above, junctions with an aluminum film as thick as 140 nm have been measured. Representative current-voltage characteristic is shown in the main panel of Fig. 2. Subgap ohmic resistance R_{sg} was extracted from experimental data as the slope of a best-fit linear regression line for quasiparticle curves in the interval from 0 to 0.2 mV where the subgap current increases linearly with *V* (the solid line in the main panel of Fig. 2). The normal-state resistance R_N was determined from a linear fit to dissipative current–voltage curves at ~ 1mV.

In the inset in Fig. 2 we compare experimental data for five different Nb/Al–AlO$_x$–Nb samples with the R_{sg}/R_N ratio values predicted for an N-I-S trilayer as well as for an S/N-I-S junction in the quasi-clean limit, this paper, and in the dirty proximity-effect limit, Refs. 7 and 8. It is evident that the novel model agrees better with experimental data, at least, for comparatively thick Al interlayers.

IV. Charge Transport in Josephson Junctions with Metallic Nanoislands Embedded into Low-Height Barriers

As was noted in the second section, superconducting junctions with low-height and, hence, comparatively thick insulating interlayers may be self-shunted by adding metallic nano-scaled drops into the barrier. First results on W-doped silicon (W:Si) interlayers in Josephson junctions formed by MoRe-alloy electrodes were published in Ref. 20. Below we present some new data and their theoretical interpretation valid at 4.2 K which corresponds to $T \approx T_c / 2$ in our samples.

In order to prevent direct transport between superconducting electrodes across regions with the transmission coefficient near unity we used an additional layer of Si with the thickness about 30-40 nm (we have found that neither junction resistance, nor superconducting characteristics were strongly affected by changing the Si thickness within this interval). In most samples, the current-voltage characteristics were non-hysteretic and all of them exhibited an excess current, a constant shift of the superconducting *I-V* curve towards that measured in the normal state at V exceeding Δ/e (the solid line in the main panel of Fig. 3). The ratio $I_c/I_{exc} \approx$ 2.4 was found for the sample shown in Fig. 3. Now we show that just this ratio can serve as an indicator of the internal structure of a weak link in Josephson junctions.

Let us calculate its value at $T \approx T_c/2$. Temperature dependences of the Josephson critical current in SNS and SIS (with strongly disordered I barrier) trilayers were presented in Ref. 12. According to Ref. 21, the excess current for an SIS sandwich can be found by doubling the corresponding result for an NIS junction

$$I_{exc} = 2\int_{-\infty}^{\infty}\left[G_{NIS}(\varepsilon) - G_{NIN}\right]\left[f(\varepsilon - eV) - f(\varepsilon)\right]d\varepsilon$$ .where

$G_{NIS}(\varepsilon)$ and G_{NIN} are related differential conductances, $f(\varepsilon)$ is the Fermi function. In the absence of the barrier, at 4.2 K we get $I_c/I_{exc} \approx$ 1.3. In the tunneling limit $D << 1$, $I_{exc} \to 0$ hence, $I_c/I_{exc} \to \infty$. Averaging related formulas with the distribution function (1) where we put $D_N = 1$, we have obtained $I_c/I_{exc} \approx 2.4$ at $T = 4.2$ K. In the inset in Fig. 3 we compare theoretical expectations and related experimental data for four samples with different dopant concentrations c_W. It follows from the data shown in the inset in Fig. 3 that the ratio I_c/I_{exc} increases with c_W as a result of the strong enhancement of the number of transport channels. But this conclusion is preliminary and should be confirmed by more detailed measurements.

Note that the product I_cR_N for the sample shown in Fig. 3 is of a comparatively large magnitude 3.8 mV. Sometimes we have even observed values several times higher. The nature of so large I_cR_N products still remains unclear.

V. Conclusion

The main aim of the paper was to show that internal shunting of Josephson junctions can be achieved using a strongly inhomogeneous insulating weak link with a bimodal transparency distribution $\rho(D)$ peaked at $D = 0$ and $D = 1$. We have discussed two possible ways to realize it, (i) an ultra-thin amorphous aluminum-oxide interlayer with very strong local fluctuations of the barrier height and thickness and (ii) comparatively thick semiconducting films with embedded metallic granulas. In the first case, the main mechanism of the charge transport is direct quantum tunneling through an inhomogeneous barrier while in the second case it is based on a quantum-percolation process including resonance trajectories with the transmission coefficient near unity.

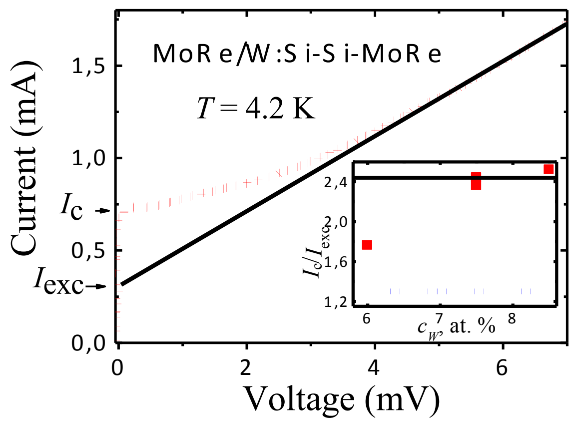

Fig. 3. The main panel: non-hysteretic *I–V* characteristic of a MoRe/W-doped Si–Si–MoRe junction with the dopant concentration 7.5 at% at 4.2 K, the thicknesses of the doped and undoped silicon interlayers were equal to 8 and 30 nm, respectively. Linear extrapolation shown by a solid line exhibits the presence of an excess current, arrows indicate critical supercurrent I_c and excess current I_{exc} values. The inset shows I_c/I_{exc} values for Si layers with different tungsten concentrations c_W; solid and dotted lines correspond to theoretical values of the ratio I_c/I_{exc} with a distribution function (1), see the text, and without any barrier, respectively, calculated at $T = T_c/2$.

We show how the predicted universal distribution function can be verified experimentally without any fitting parameters and analyze some old and new experimental data from this perspective. We compare theoretical results for the first-type Josephson devices with the data measured for Nb/Al–AlO_x–Nb four-layered junctions with more than 100 nm-thick aluminum interlayers and present some preliminary data obtained on MoRe/W:Si-Si-MoRe devices. We believe that these results can form a base for novel four-layered Josephson junctions with enhanced superconducting properties and, at the same time, well-separated metallic electrodes.

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